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DOI:

[10.1109/TSMC.2016.2598785](https://doi.org/10.1109/TSMC.2016.2598785)

Document Version

Peer reviewed version

[Link to publication record in King's Research Portal](#)

Citation for published version (APA):

Wei, Y., Qiu, J., Shi, P., & Lam, H-K. (2017). A New Design of H-Infinity Piecewise Filtering for Discrete-Time Nonlinear Time-Varying Delay Systems via T-S Fuzzy Affine Models. *Ieee Transactions On Systems Man And Cybernetics Part A-Systems And Humans*, 47(8), 2034-2047. [7556378].
<https://doi.org/10.1109/TSMC.2016.2598785>

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A New Design of H-infinity Piecewise Filtering for Discrete-Time Nonlinear Time-Varying Delay Systems via T-S Fuzzy Affine Models

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Abstract—This paper proposes a novel delay-dependent approach to the piecewise-affine H-infinity filter design for discrete-time state-delayed nonlinear systems. The nonlinear plant is expressed by a Takagi-Sugeno fuzzy-affine model and the state delay is considered to be time-varying with available lower and upper bounds. The purpose is to design an admissible filter that guarantees the asymptotic stability of the resulting filtering error system with a prescribed disturbance attenuation level in an H-infinity sense. By applying a new piecewise-fuzzy Lyapunov-Krasovskii functional, combined with a novel summation inequality, improved reciprocally convex inequality and S-procedure, the H-infinity performance analysis criterion is firstly developed for the FES. Furthermore, the filter synthesis is carried out by some elegant convexification techniques. Finally, simulation examples are employed to confirm the effectiveness and less conservatism of the proposed methods.

Index Terms—T-S fuzzy-affine systems, New summation inequality, Filter design, Time-varying delay.

I. INTRODUCTION

Takagi-Sugeno (T-S) model-based fuzzy control approach has been widely investigated and employed to synthesize many complicated nonlinear systems during the past years [1]–[5]. In essence, a T-S fuzzy model comprises of a family of local linear models in various premise variable subspaces, which are connected in terms of fuzzy-membership functions. It has been shown that T-S fuzzy models are well appropriate to universally approximate any smooth nonlinear functions in any compact set with any degree of accuracy [6]–[22]. Therefore, substantial efforts have been dedicated to systematically solve the analysis, controller synthesis, and filter design of fuzzy systems. For the recent development in this respect, please track the survey paper [23] and [24] and the literature therein.

This work was supported in part by the National Natural Science Foundation of China (61503091, 61374031, 61522306, 61573112, U1509217), the Australian Research Council (DP140102180, LP140100471), the China Postdoctoral Science Foundation (2016T90270, 2015M570282), the Postdoctoral Science Foundation of Heilongjiang Province (LBH-Z14056), and the Harbin Special Funds for Technological Innovation Research (2014RFQXJ067).

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As the dual problem of control design, the topic of state estimation prevails in signal processing and control disciplines. Correspondingly, different scenarios on state estimation for various classes of dynamic systems have been developed [25]–[29]. Specifically, in the single/common Lyapunov function/functional framework, many useful results on state estimation for fuzzy-dynamic systems with or without state-delay have been presented in [30]–[35]. To reduce the analysis and synthesis conservatism, some studies on robust filter synthesis for fuzzy-dynamic systems in the piecewise or fuzzy-basis-dependent Lyapunov functions framework have been reported in [27]–[30]. It has been demonstrated that piecewise/fuzzy-Lyapunov functions are with much more plentiful classes of Lyapunov function candidates than a single Lyapunov function candidate and, therefore, feature the capabilities to address the analysis and design for a boarder class of fuzzy-dynamic systems [27].

Additionally, it is also found that time delay exists in many practical systems, such as networked control systems, mechanical systems, and power systems. It has been shown that the appearance of time delay may incur nonminimum phase, poor performance of control systems, even instability. Thus, there have been many results involving delay-dependent and delay-independent types on the analysis and design for T-S fuzzy model-based time-delay systems and it has been validated that the delay-dependent conditions generally reflect the reality better [29], [31], [36]–[39]. In general, most of the approaches entail the construction of Lyapunov-Krasovskii functionals (LKFs) [31], and the employment of some comparatively tight techniques, such as Jensen's inequality [39], free-weighting matrix approach [29], delay-partitioning approach [40], input-output approach [41], or reciprocally convex approach [42], to derive numerically tractable stability analysis criteria for the time-delayed fuzzy systems. In retrospect, based on the free-weighting matrix approach, the authors in [29] investigated the problem of robust filter synthesis for fuzzy time-delay systems; By applying the delay-partitioning approach, the robust \mathcal{H}_∞ filter analysis and synthesis problems for continuous-time fuzzy systems with state-delay were discussed in [40]; In [41], the authors dealt with the l_2 - l_∞ filtering problem for delayed fuzzy systems in an input-output framework; In [42], by utilizing the reciprocally convex approach, the authors considered the reliable filter design problem for discrete-time T-S fuzzy time-delay systems in a strict dissipativity sense. It is worth mentioning that the input-output approach relies es-

sentially on the application of the model transformation, which is directly related to the selection of the LKFs. In contrast, the reciprocally convex approach lies mainly on the bound of some crossing terms which appears in the difference/derivative of the Lyapunov functional. It has been shown that these choices of LKFs and over-bounding techniques inevitably induce some degree of conservatism. Therefore, the approaches developed to the filter design of time-delay fuzzy systems still have room for improvement in terms of conservatism. Moreover, it is also worth pointing out that most of the existing filter synthesis results were presented merely for T-S fuzzy linear systems, yet it has been found that T-S fuzzy dynamic systems with offset terms are characterized with much improved function approximation ability. All the issues above-mentioned reveal that the piecewise-affine filter synthesis for T-S fuzzy-affine systems with time-delay has not been fully studied and remains to be open and challenging.

Based on the aforementioned statements, this paper will concern the delay-dependent piecewise-affine \mathcal{H}_∞ filter synthesis problem for discrete-time T-S fuzzy-affine systems subject to time-varying delay. Specifically, by constructing an appropriate piecewise-fuzzy LKF, together with a novel summation inequality, improved reciprocally convex approach and S-procedure, a new bounded real lemma will be firstly proposed for the filtering error system (FES), and then the piecewise-affine filter synthesis will be developed by some elegant convexification techniques. It will be shown that the desired filter gains can be obtained in a convex optimization framework. Finally, simulation examples will be presented to clarify the effectiveness and less conservatism of the developed design approaches.

Notations. The notations utilized are standard. \mathbb{S}^n represents the set of $n \times n$ real symmetric and positive definite matrices; $\text{Sym}\{A\}$ denotes the shorthand notation for $A + A^T$; $l_2[0, \infty)$ refers to the space of square summable infinite sequence and for $w = \{w(k)\} \in l_2[0, \infty)$, its norm is characterized by $\|w\|_2 = \sqrt{\sum_{k=0}^{\infty} \|w(k)\|^2}$.

II. MODEL DESCRIPTION AND PROBLEM FORMULATION

Consider a class of nonlinear systems, which can be approximated by the T-S fuzzy-dynamic model characterized with IF-THEN rules. Following the idea of [43], a general discrete-time T-S fuzzy-affine dynamic model can be expressed as the following form,

Plant Rule \mathcal{F}^l : IF $\xi_1(k)$ is F_1^l and $\xi_2(k)$ is F_2^l and \dots and $\xi_g(k)$ is F_g^l , THEN

$$\begin{cases} x(k+1) = A_l x(k) + A_{dl} x(k-d(k)) + a_l + B_l w(k) \\ y(k) = C_l x(k) + C_{dl} x(k-d(k)) + D_l w(k) \\ z(k) = L_l x(k) + L_{dl} x(k-d(k)) \\ x(k) = \phi(k), \quad -d_2 \leq k \leq 0, \quad l \in \mathcal{L} := \{1, 2, \dots, M\} \end{cases} \quad (1)$$

where \mathcal{F}^l represents the l -th fuzzy inference rule; M stands for the number of inference rules; $F_\nu^l (\nu = 1, 2, \dots, g)$ are fuzzy sets; $x(k) \in \mathbb{R}^{n_x}$ represents the system state; $y(k) \in \mathbb{R}^{n_y}$ denotes the measurement output; $z(k) \in \mathbb{R}^{n_z}$ denotes the signal to be estimated; $w(k) \in \mathbb{R}^{n_w}$ denotes the exogenous disturbance input belonging to $l_2[0, \infty)$; $\xi(k) :=$

$[\xi_1(k), \xi_2(k), \dots, \xi_g(k)]$ are the premise variables; $d(k)$ denotes the time-varying delay, which is a positive integer function and satisfies the following assumption:

$$d_1 \leq d(k) \leq d_2, \quad (2)$$

where d_1 and d_2 are two constant positive integers referring to the minimum and maximum time-delay, respectively. $\phi(k)$ is the initial condition on $[-d_2, 0]$; $(A_l, A_{dl}, a_l, B_l, C_l, C_{dl}, D_l, L_l, L_{dl})$ refers to the system matrices of the l -th local model.

Let $\mu_l[\xi(k)]$ be the normalized fuzzy-basis function of F^l where $F^l := \prod_{\nu=1}^g F_\nu^l$ and

$$\mu_l[\xi(k)] := \frac{\prod_{\nu=1}^g \mu_{l\nu}[\xi_\nu(k)]}{\sum_{p=1}^M \prod_{\nu=1}^g \mu_{p\nu}[\xi_\nu(k)]} \geq 0, \quad \sum_{l=1}^M \mu_l[\xi(k)] = 1 \quad (3)$$

where $\mu_{l\nu}[\xi_\nu(k)]$ is the grade of membership of $\xi_\nu(k)$ in F_ν^l . To simplify the notations, we subsequently specify μ_l as $\mu_l[\xi(k)]$.

By utilizing a singleton fuzzifier, product fuzzy inference and center-average defuzzifier, we end up with the following global T-S fuzzy dynamic model,

$$\begin{cases} x(k+1) = A(\mu)x(k) + A_d(\mu)x(k-d(k)) \\ \quad + a(\mu) + B(\mu)w(k) \\ y(k) = C(\mu)x(k) + C_d(\mu)x(k-d(k)) + D(\mu)w(k) \\ z(k) = L(\mu)x(k) + L_d(\mu)x(k-d(k)) \end{cases} \quad (4)$$

where

$$\chi(\mu) := \sum_{l=1}^M \mu_l \chi_l, \quad \chi \in \{A, A_d, a, B, C, C_d, D, L, L_d\}. \quad (5)$$

This paper tackles the delay-dependent \mathcal{H}_∞ filtering problem for the fuzzy-affine dynamic model (4) in the framework of piecewise-fuzzy Lyapunov functionals. Similar to [44] and [45], we can further rewrite the system (4) as a piecewise-fuzzy-affine system with the form,

$$\begin{cases} x(k+1) = \mathcal{A}_i x(k) + \mathcal{A}_{di} x(k-d(k)) + \mathbf{a}_i + \mathcal{B}_i w(k) \\ y(k) = \mathcal{C}_i x(k) + \mathcal{C}_{di} x(k-d(k)) + \mathcal{D}_i w(k) \\ z(k) = \mathcal{L}_i x(k) + \mathcal{L}_{di} x(k-d(k)) \\ x(k) = \phi(k), \quad -d_2 \leq k \leq 0, \quad \xi(k) \in \mathcal{S}_i, \quad i \in \mathcal{I} \end{cases} \quad (6)$$

where $\{\mathcal{S}_i\}_{i \in \mathcal{I}}$ and \mathcal{I} , respectively, stand for the state-space division and the set of subspace indexes, and

$$\begin{cases} \mathcal{A}_i := \sum_{s \in \mathcal{S}(i)} \mu_s \mathcal{A}_s, \quad \mathcal{A}_{di} := \sum_{s \in \mathcal{S}(i)} \mu_s \mathcal{A}_{ds}, \\ \mathbf{a}_i := \sum_{s \in \mathcal{S}(i)} \mu_s \mathbf{a}_s, \quad \mathcal{B}_i := \sum_{s \in \mathcal{S}(i)} \mu_s \mathcal{B}_s, \\ \mathcal{C}_i := \sum_{s \in \mathcal{S}(i)} \mu_s \mathcal{C}_s, \quad \mathcal{C}_{di} := \sum_{s \in \mathcal{S}(i)} \mu_s \mathcal{C}_{ds}, \\ \mathcal{D}_i := \sum_{s \in \mathcal{S}(i)} \mu_s \mathcal{D}_s, \quad \mathcal{L}_i := \sum_{s \in \mathcal{S}(i)} \mu_s \mathcal{L}_s, \\ \mathcal{L}_{di} := \sum_{s \in \mathcal{S}(i)} \mu_s \mathcal{L}_{ds} \end{cases} \quad (7)$$

with $0 < \mu_s[\xi(k)] \leq 1$, $\sum_{s \in \mathcal{S}(i)} \mu_s[\xi(k)] = 1$. For each subspace \mathcal{S}_i , $i \in \mathcal{I}$, the set $\mathcal{S}(i)$ contains the indexes for the local system modes utilized in the interpolation within that subspace. Notice that $\mathcal{S}(i)$ in a crisp subspace only involves one subsystem.

To address the piecewise-affine filter synthesis problem of fuzzy-affine system (6) in the piecewise-fuzzy Lyapunov function framework, we also specify a new set Ω to involve all possible region switchings

$$\Omega := \{(i, j) | \xi(k) \in \mathcal{S}_i, \xi(k+1) \in \mathcal{S}_j, i, j \in \mathcal{I}\}. \quad (8)$$

Specifically, in the context of $j = i$ with $(i, j) \in \Omega$, the state evolutions occur in the same subspace \mathcal{S}_i at the time k . Otherwise, the states will jump from the subspace \mathcal{S}_i to \mathcal{S}_j at that time.

Notice that the partitions induced by fuzzy rules are polyhedral regions in the state-space. Meantime, the intersection of several polyhedra is inherently a polyhedron, and the cells of the resulting partition are thus polyhedra. Interestingly, an ellipsoid \mathcal{E}_i can be well utilized to outer approximate each polyhedral region \mathcal{S}_i [44], [45]. Specifically, suppose that there exist matrices Q_i and q_i satisfying

$$\mathcal{S}_i \subseteq \mathcal{E}_i, \mathcal{E}_i = \{x | \|Q_i x + q_i\| \leq 1\}. \quad (9)$$

Admittedly, this outer approximation is very useful in the context that \mathcal{S}_i are slab regions, as in the case, the parameters Q_i and q_i are guaranteed to exist, and the covering is accurate, i.e., $\mathcal{S}_i \subseteq \mathcal{E}_i$ and $\mathcal{E}_i \subseteq \mathcal{S}_i$ [45]. Especially, when the polyhedral regions \mathcal{S}_i are slabs in the following form,

$$\mathcal{S}_i = \{x | \varphi_{1i} \leq \theta_i^T x \leq \varphi_{2i}\} \quad (10)$$

where $\varphi_{1i}, \varphi_{2i} \in \mathbb{R}$, each slab region can correspondingly be precisely characterized by a degenerate ellipsoid in the form of (9) with

$$Q_i = \frac{2\theta_i^T}{\varphi_{2i} - \varphi_{1i}}, q_i = -\frac{\varphi_{2i} + \varphi_{1i}}{\varphi_{2i} - \varphi_{1i}}. \quad (11)$$

From (9), we immediately obtain the following condition for each ellipsoid region,

$$\begin{bmatrix} x(k) \\ 1 \end{bmatrix}^T \begin{bmatrix} Q_i^T Q_i & Q_i^T q_i \\ q_i^T Q_i & q_i^T q_i - 1 \end{bmatrix} \begin{bmatrix} x(k) \\ 1 \end{bmatrix} \leq 0, \quad i \in \mathcal{I}. \quad (12)$$

Let the indexes of the partitioned regions be divided into two classes $\mathcal{I} = \mathcal{I}_0 \cup \mathcal{I}_1$, where \mathcal{I}_0 represents the index set of regions with $q_i^T q_i - 1 \leq 0$ which covers the origin, and \mathcal{I}_1 denotes the index set of regions otherwise.

For the fuzzy-affine dynamic system (6), we synthesize the following *piecewise-affine* filter,

$$\begin{cases} \hat{x}(k+1) = A_{fi}\hat{x}(k) + a_{fi} + B_{fi}y(k) \\ \hat{z}(k) = C_{fi}\hat{x}(k) + D_{fi}y(k), \xi(k) \in \mathcal{S}_i, i \in \mathcal{I} \end{cases} \quad (13)$$

where $\hat{x}(k)$ is the filter state; $\hat{z}(k)$ is the estimation of $z(k)$; and $A_{fi} \in \mathbb{R}^{n_x \times n_x}$, $B_{fi} \in \mathbb{R}^{n_x \times n_y}$, $C_{fi} \in \mathbb{R}^{n_z \times n_x}$, $D_{fi} \in \mathbb{R}^{n_z \times n_y}$, $i \in \mathcal{I}$, and $a_{fi} \in \mathbb{R}^{n_x \times 1}$, $\forall i \in \mathcal{I}_1$, are piecewise filter gains to be synthesized. It is noted that when $a_{fi} \equiv 0$, $i \in \mathcal{I}$, then (13) reduces to a traditional piecewise-linear filter.

Augmenting the state vector $\bar{x}(k) := [x^T(k) \quad \hat{x}^T(k)]^T$, and $\bar{z}(k) := z(k) - \hat{z}(k)$, we immediately acquire the following FES,

$$\begin{cases} \bar{x}(k+1) = \bar{\mathcal{A}}_i \bar{x}(k) + \bar{\mathcal{A}}_{di} H \bar{x}(k-d(k)) + \bar{\mathbf{a}}_i + \bar{\mathcal{B}}_i w(k) \\ \bar{z}(k) = \bar{\mathcal{L}}_i \bar{x}(k) + \bar{\mathcal{L}}_{di} H \bar{x}(k-d(k)) + \bar{\mathcal{D}}_i w(k) \\ \bar{x}(k) = [\phi^T(k) \quad \mathbf{0}]^T, k \in [-d_2, 0], \end{cases} \quad (14)$$

where

$$\begin{cases} \bar{\mathcal{A}}_i := \begin{bmatrix} \mathcal{A}_i & \mathbf{0} \\ B_{fi}C_i & A_{fi} \end{bmatrix}, \bar{\mathcal{A}}_{di} := \begin{bmatrix} \mathcal{A}_{di} \\ B_{fi}C_{di} \end{bmatrix}, \\ \bar{\mathcal{B}}_i := \begin{bmatrix} \mathcal{B}_i \\ B_{fi}D_i \end{bmatrix}, \bar{\mathbf{a}}_i := \begin{bmatrix} \mathbf{a}_i \\ a_{fi} \end{bmatrix}, \\ \bar{\mathcal{L}}_i := \begin{bmatrix} \mathcal{L}_i - D_{fi}C_i & -C_{fi} \end{bmatrix}, \bar{\mathcal{L}}_{di} := \mathcal{L}_{di} - D_{fi}C_{di}, \\ \bar{\mathcal{D}}_i := -D_{fi}D_i, H := [\mathbf{I} \quad \mathbf{0}_{n_x \times n_x}]. \end{cases} \quad (15)$$

Then, the piecewise-affine filtering problem to be addressed in the paper is elaborated as follows.

Robust \mathcal{H}_∞ Piecewise-Affine Filter Design Problem. Considering the fuzzy-affine system (6), synthesize a piecewise-affine filter (13) to estimate the signal $z(k)$ such that the FES (14) can achieve the asymptotic stability and the induced l_2 -norm of the operator mapping from exogenous input w to the filtering error \bar{z} is less than γ under zero initial conditions,

$$\mathcal{H}_\infty^{\bar{z}w} := \sup \frac{\|\bar{z}\|_2}{\|w\|_2} < \gamma \quad (16)$$

for all nonzero $w \in l_2[0, \infty)$.

The following lemmas will be utilized in deriving performance analysis condition of the system (14).

Lemma 2.1. [46] For a given matrix $Z \in \mathbb{S}^n$, integers $d_1 \leq d_2$, and vector function $\eta : \{d_1, d_1+1, \dots, d_2\} \rightarrow \mathbb{R}^n$, the following inequality holds,

$$d \sum_{s=-d_2+1}^{d_1} \eta^T(s) Z \eta(s) \geq \begin{bmatrix} \varsigma_1 \\ \varsigma_2 \end{bmatrix}^T \begin{bmatrix} Z & \mathbf{0} \\ \mathbf{0} & 3(\frac{d+1}{d-1})Z \end{bmatrix} \begin{bmatrix} \varsigma_1 \\ \varsigma_2 \end{bmatrix}, \quad (17)$$

where

$$\begin{cases} \eta(s) := x(s+1) - x(s), d = d_2 - d_1, \\ \varsigma_1 := x(-d_1) - x(-d_2), \\ \varsigma_2 := x(-d_1) + x(-d_2) - \frac{2}{d+1} \sum_{s=-d_2}^{-d_1} x(s). \end{cases}$$

In some cases, to reduce the computational burden induced by the factor $\frac{d+1}{d-1}$, an alternative lemma without this factor is introduced as follows.

Lemma 2.2. [46] For a given matrix $Z \in \mathbb{S}^n$, integers $d_1 \leq d_2$, and vector function $\eta : \{d_1, d_1+1, \dots, d_2\} \rightarrow \mathbb{R}^n$, the following inequality holds,

$$d \sum_{s=-d_2+1}^{d_1} \eta^T(s) Z \eta(s) \geq \begin{bmatrix} \varsigma_1 \\ \varsigma_2 \end{bmatrix}^T \begin{bmatrix} Z & \mathbf{0} \\ \mathbf{0} & 3Z \end{bmatrix} \begin{bmatrix} \varsigma_1 \\ \varsigma_2 \end{bmatrix},$$

where $\eta(s)$, ς_1 and ς_2 are defined in Lemma 2.1.

III. PIECEWISE-AFFINE \mathcal{H}_∞ FILTER ANALYSIS AND SYNTHESIS

In this section, by constructing a suitable piecewise-fuzzy LKF, together with a new summation inequality, improved reciprocally convex inequality and S-procedure, we shall firstly present the \mathcal{H}_∞ performance analysis criterion for the FES (14). Then via some elegant convexification procedures, the piecewise-affine filter synthesis will be carried out.

A. Delay-Dependent \mathcal{H}_∞ Performance Analysis

Theorem 3.1. The FES in (14) is stable with an \mathcal{H}_∞ performance γ , if there exist matrices $P_i(\mu) \in \mathbb{S}^{4n_x}$, $\{R_1, R_2, Z_1, Z_2\} \in \mathbb{S}^{n_x}$, $X \in \mathbb{R}^{2n_x \times 2n_x}$, $\bar{G}_{1i} \in \mathbb{R}^{(10n_x+n_w) \times 2n_x}$, $i \in \mathcal{I}$, and $\bar{G}_{2i} \in \mathbb{R}^{1 \times 2n_x}$, and scalar $\lambda_i < 0$, $i \in \mathcal{I}_1$, guaranteeing that the subsequent matrix inequalities hold,

$$\begin{aligned} & \text{Sym}\{\mathcal{G}_i \mathcal{A}_i\} + \Lambda_1 P_j(\mu^+) \Lambda_1^T - \Lambda_2 P_i(\mu) \Lambda_2^T + \Lambda_3 \mathcal{R} \Lambda_3^T \\ & + \Lambda_4 \mathcal{Z}_0 \Lambda_4^T - \Lambda_5 \mathcal{Z}_1 \Lambda_5^T - \Lambda_6(d(k)) \mathcal{Z}_2 \Lambda_6^T(d(k)) \\ & + \bar{\lambda}_i \Lambda_7 \mathcal{Q}_i \Lambda_7^T + \mathcal{L}_i^T \mathcal{L}_i - \gamma^2 E_9 E_9^T < 0, \quad (i, j) \in \Omega, \quad (18) \end{aligned}$$

where $\mu := [\mu_1(\xi(k)), \dots, \mu_M(\xi(k))]$, $\mu^+ := [\mu_1(\xi(k+1)), \dots, \mu_M(\xi(k+1))]$, and

$$\left\{ \begin{aligned} \Lambda_1 &:= \begin{bmatrix} E_1 & E_6 - E_4 & E_7 + E_8 - E_3 - E_5 \end{bmatrix}, \\ \Lambda_2 &:= \begin{bmatrix} E_2 & E_6 - E_2 H^T & E_7 + E_8 - E_3 - E_4 \end{bmatrix}, \\ \Lambda_3 &:= \begin{bmatrix} E_2 & E_4 & E_5 \end{bmatrix}, \quad \Lambda_4 := \begin{bmatrix} E_1 & E_2 \end{bmatrix}, \\ \Lambda_5 &:= \begin{bmatrix} E_2 H^T - E_4 & E_2 H^T + E_4 - \frac{2}{d_1+1} E_6 \end{bmatrix}, \\ \Lambda_6(d(k)) &:= \begin{bmatrix} E_4 - E_3 & E_4 + E_3 - \frac{2}{d(k)-d_1+1} E_7 \\ E_3 - E_5 & E_3 + E_5 - \frac{2}{d_2-d(k)+1} E_8 \end{bmatrix}, \\ \Lambda_7 &:= \begin{bmatrix} E_2 & E_{10} \end{bmatrix}, \\ \mathcal{R} &:= \text{diag}\{H^T R_1 H, R_2 - R_1, -R_2\}, \\ \mathcal{Z}_0 &:= \begin{bmatrix} H^T(d_1^2 Z_1 + d^2 Z_2) H & * \\ -H^T(d_1^2 Z_1 + d^2 Z_2) H & H^T(d_1^2 Z_1 + d^2 Z_2) H \end{bmatrix}, \\ \mathcal{Z}_1 &:= \text{diag}\{Z_1, 3(\frac{d+1}{d-1})Z_1\}, \\ \mathcal{G}_i &:= \bar{G}_{1i}, \quad \bar{\lambda}_i = 0, \\ \mathcal{A}_i &:= \begin{bmatrix} -\mathbf{I} & \bar{\mathcal{A}}_i & \bar{\mathcal{A}}_{di} & \mathbf{0}_{2n_x \times 5n_x} & \bar{\mathcal{B}}_i \end{bmatrix}, \\ \mathcal{L}_i &:= \begin{bmatrix} \mathbf{0}_{n_z \times 2n_x} & \bar{\mathcal{L}}_i & \bar{\mathcal{L}}_{di} & \mathbf{0}_{n_z \times 5n_x} & \bar{\mathcal{D}}_i \end{bmatrix}, \\ E_\kappa &:= \begin{bmatrix} \underbrace{\mathbf{0} \cdots \mathbf{0}}_{\kappa-1} & \mathbf{I}_{\sigma(\kappa)} & \underbrace{\mathbf{0} \cdots \mathbf{0}}_{9-\kappa} \end{bmatrix}^T, \\ &\quad \in \mathbb{R}^{(10n_x+n_w) \times \sigma(\kappa)}, \quad \kappa = 1, \dots, 9, \\ &\quad \text{if } i \in \mathcal{I}_0, \\ \mathcal{G}_i &:= \begin{bmatrix} \bar{G}_{1i}^T & \bar{G}_{2i}^T \end{bmatrix}^T, \quad \bar{\lambda}_i = \lambda_i, \\ \mathcal{A}_i &:= \begin{bmatrix} -\mathbf{I} & \bar{\mathcal{A}}_i & \bar{\mathcal{A}}_{di} & \mathbf{0}_{2n_x \times 5n_x} & \bar{\mathcal{B}}_i & \bar{\mathbf{a}}_i \end{bmatrix}, \\ \mathcal{L}_i &:= \begin{bmatrix} \mathbf{0}_{n_z \times 2n_x} & \bar{\mathcal{L}}_i & \bar{\mathcal{L}}_{di} & \mathbf{0}_{n_z \times 5n_x} & \bar{\mathcal{D}}_i & \mathbf{0}_{n_z \times 1} \end{bmatrix}, \\ E_\kappa &:= \begin{bmatrix} \underbrace{\mathbf{0} \cdots \mathbf{0}}_{\kappa-1} & \mathbf{I}_{\sigma(\kappa)} & \underbrace{\mathbf{0} \cdots \mathbf{0}}_{10-\kappa} \end{bmatrix}^T, \\ &\quad \in \mathbb{R}^{(10n_x+n_w+1) \times \sigma(\kappa)}, \quad \kappa = 1, \dots, 10, \\ &\quad \text{if } i \in \mathcal{I}_1, \\ \mathcal{Z}_2 &:= \begin{bmatrix} \text{diag}\{Z_2, 3Z_2\} & * \\ X^T & \text{diag}\{Z_2, 3Z_2\} \end{bmatrix}, \\ \mathcal{Q}_i &:= \begin{bmatrix} H^T Q_i^T Q_i H & * \\ q_i^T Q_i H & q_i^T q_i - 1 \end{bmatrix}, \end{aligned} \right\}, \quad (19)$$

with

$$\sigma(\kappa) := \begin{cases} 2n_x, & \kappa = 1, 2, \\ n_x, & \kappa = 3, \dots, 8, \\ n_w, & \kappa = 9, \\ 1, & \kappa = 10. \end{cases} \quad (20)$$

Proof. It is evident that the condition (18) for $i \in \mathcal{I}_0$ is a particular case of (18) for $i \in \mathcal{I}_1$ in the context of $a_l = 0$ and the S-procedure is not utilized. For simplicity, here we only present the proof of the more complicated condition (18) with $i \in \mathcal{I}_1$.

Construct the following piecewise-fuzzy LKF for the FES in (14),

$$V(\bar{x}(k), k) := \sum_{\varrho=1}^3 V_\varrho(\bar{x}(k), k), \quad (21)$$

with

$$\left\{ \begin{aligned} V_1(\bar{x}(k), k) &:= \bar{x}^T(k) P_i(\mu) \bar{x}(k), \\ V_2(\bar{x}(k), k) &:= \sum_{s=k-d_1}^{k-1} \bar{x}^T(s) H^T R_1 H \bar{x}(s) \\ &\quad + \sum_{s=k-d_2}^{k-d_1-1} \bar{x}^T(s) H^T R_2 H \bar{x}(s), \\ V_3(\bar{x}(k), k) &:= d_1 \sum_{s=k-d_1}^{-1} \sum_{u=k+s}^{k-1} \bar{\eta}^T(u) H^T Z_1 H \bar{\eta}(u) \\ &\quad + d \sum_{s=-d_2}^{-d_1-1} \sum_{u=k+s}^{k-1} \bar{\eta}^T(u) H^T Z_2 H \bar{\eta}(u), \end{aligned} \right. \quad (22)$$

where

$$\left\{ \begin{aligned} \bar{x}(k) &:= \begin{bmatrix} \bar{x}^T(k) & \sum_{s=k-d_1}^{k-1} x^T(s) & \sum_{s=k-d_2}^{k-d_1-1} x^T(s) \end{bmatrix}^T, \\ \bar{\eta}(u) &:= \bar{x}(u+1) - \bar{x}(u) \end{aligned} \right. \quad (23)$$

and $P_i(\mu) \in \mathbb{S}^{4n_x}$, $\{R_1, R_2, Z_1, Z_2\} \in \mathbb{S}^{n_x}$.

Then, considering the piecewise-fuzzy LKF in (21)-(23), the FES (14) is stable with an \mathcal{H}_∞ performance γ under zero initial conditions and any nonzero $w(k) \in l_2[0, \infty)$, if one can show

$$\Delta V(\bar{x}(k), k) + \bar{z}^T(k) \bar{z}(k) - \gamma^2 w^T(k) w(k) < 0, \quad (24)$$

where $\Delta V(\bar{x}(k), k) := V(\bar{x}(k+1), k+1) - V(\bar{x}(k), k)$.

In the following derivations, our purpose is to attain an upper bound of $\Delta V(\bar{x}(k), k)$ according to the augmented vector

$$\left\{ \begin{aligned} \zeta(k) &:= \begin{bmatrix} \bar{x}^T(k+1) & \bar{x}^T(k) & \bar{x}^T(k-d(k)) & x^T(k-d_1) & x^T(k-d_2) & \vartheta_1^T(k) & \vartheta_2^T(k) & \vartheta_3^T(k) & w^T(k) & 1 \end{bmatrix}^T, \\ \vartheta_1(k) &:= \sum_{s=k-d_1}^k x(s), \\ \vartheta_2(k) &:= \sum_{s=k-d(k)}^{k-d_1} x(s), \\ \vartheta_3(k) &:= \sum_{s=k-d_2}^{k-d(k)} x(s). \end{aligned} \right. \quad (25)$$

Define $E_\kappa := \begin{bmatrix} \underbrace{\mathbf{0} \cdots \mathbf{0}}_{\kappa-1} & \mathbf{I}_{\sigma(\kappa)} & \underbrace{\mathbf{0} \cdots \mathbf{0}}_{10-\kappa} \end{bmatrix}^T \in \mathbb{R}^{(10n_x+n_w+1) \times \sigma(\kappa)}$, $1 \leq \kappa \leq 10$, where

$$\sigma(\kappa) := \begin{cases} 2n_x, & \kappa = 1, 2, \\ n_x, & \kappa = 3, \dots, 8, \\ n_w, & \kappa = 9, \\ 1, & \kappa = 10. \end{cases} \quad (26)$$

It is obvious that $\bar{x}(k+1) = E_1^T \zeta(k)$, $x(k-d(k)) = E_3^T \zeta(k)$, $w(k) = E_9^T \zeta(k)$, and $1 = E_{10}^T \zeta(k)$.

Performing the difference of $V(\bar{x}(k), k)$ along the trajectory of FES (14), leads to

$$\begin{aligned} \Delta V_1 &= V_1(\bar{x}(k+1), k+1) - V_1(\bar{x}(k), k) \\ &= \begin{bmatrix} \bar{x}(k+1) \\ \vartheta_1(k) - x(k-d_1) \\ \vartheta_2(k) + \vartheta_3(k) - x(k-d(k)) - x(k-d_2) \end{bmatrix}^T \\ &\quad \times P_j(\mu^+)(*) \\ &\quad - \begin{bmatrix} \bar{x}(k) \\ \vartheta_1(k) - H\bar{x}(k) \\ \vartheta_2(k) + \vartheta_3(k) - x(k-d(k)) - x(k-d_1) \end{bmatrix}^T \\ &\quad \times P_i(\mu)(*) \\ &= \zeta^T(k)(\Lambda_1 P_j(\mu^+) \Lambda_1^T - \Lambda_2 P_i(\mu) \Lambda_2^T) \zeta(k), \\ \Delta V_2 &= V_2(\bar{x}(k+1), k+1) - V_2(\bar{x}(k), k) \\ &= \bar{x}^T(k) H^T R_1 H \bar{x}(k) \\ &\quad + x^T(k-d_1)(R_2 - R_1)x(k-d_1) \\ &\quad - x^T(k-d_2)R_2 x(k-d_2) \\ &= \zeta^T(k) \Lambda_3 \mathcal{R} \Lambda_3^T \zeta(k), \\ \Delta V_3 &= V_3(\bar{x}(k+1), k+1) - V_3(\bar{x}(k), k) \\ &= \bar{\eta}^T(k) H^T (d_1^2 Z_1 + d^2 Z_2) H \bar{\eta}(k) \\ &\quad - d_1 \sum_{s=k-d_1}^{k-1} \bar{\eta}^T(s) H^T Z_1 H \bar{\eta}(s) \\ &\quad - d \sum_{s=k-d(k)}^{k-d_1-1} \bar{\eta}^T(s) H^T Z_2 H \bar{\eta}(s) \\ &\quad - d \sum_{s=k-d_2}^{k-d(k)-1} \bar{\eta}^T(s) H^T Z_2 H \bar{\eta}(s), \end{aligned} \quad (27)$$

$$\begin{aligned} \Delta V_2 &= V_2(\bar{x}(k+1), k+1) - V_2(\bar{x}(k), k) \\ &= \bar{x}^T(k) H^T R_1 H \bar{x}(k) \\ &\quad + x^T(k-d_1)(R_2 - R_1)x(k-d_1) \\ &\quad - x^T(k-d_2)R_2 x(k-d_2) \\ &= \zeta^T(k) \Lambda_3 \mathcal{R} \Lambda_3^T \zeta(k), \\ \Delta V_3 &= V_3(\bar{x}(k+1), k+1) - V_3(\bar{x}(k), k) \\ &= \bar{\eta}^T(k) H^T (d_1^2 Z_1 + d^2 Z_2) H \bar{\eta}(k) \\ &\quad - d_1 \sum_{s=k-d_1}^{k-1} \bar{\eta}^T(s) H^T Z_1 H \bar{\eta}(s) \\ &\quad - d \sum_{s=k-d(k)}^{k-d_1-1} \bar{\eta}^T(s) H^T Z_2 H \bar{\eta}(s) \\ &\quad - d \sum_{s=k-d_2}^{k-d(k)-1} \bar{\eta}^T(s) H^T Z_2 H \bar{\eta}(s), \end{aligned} \quad (28)$$

where $\mu := [\mu_1(\xi(k)), \dots, \mu_M(\xi(k))]$, $\mu^+ := [\mu_1(\xi(k+1)), \dots, \mu_M(\xi(k+1))]$, $d = d_2 - d_1$, and

$$\begin{cases} \mathcal{R} := \text{diag}\{H^T R_1 H, R_2 - R_1, -R_2\}, \\ \Lambda_1 := \begin{bmatrix} E_1 & E_6 - E_4 & E_7 + E_8 - E_3 - E_5 \end{bmatrix}, \\ \Lambda_2 := \begin{bmatrix} E_2 & E_6 - E_2 H^T & E_7 + E_8 - E_3 - E_4 \end{bmatrix}, \\ \Lambda_3 := \begin{bmatrix} E_2 & E_4 & E_5 \end{bmatrix}, \end{cases} \quad (30)$$

and $\zeta(k)$, $\vartheta_1(k)$, $\vartheta_2(k)$, $\vartheta_3(k)$, and $E_\kappa, \kappa = 1, \dots, 8$ are defined in (25) and (26), respectively.

Using Lemma 2.1 to the second term in the RHS of equation (29), one has

$$\begin{aligned} d_1 \sum_{s=k-d_1}^{k-1} \bar{\eta}^T(s) H^T Z_1 H \bar{\eta}(s) \\ \geq \begin{bmatrix} x(k) - x(k-d_1) \\ x(k) + x(k-d_1) - \frac{2}{d_1+1} \vartheta_1(k) \end{bmatrix}^T \\ \times \text{diag}\{Z_1, 3 \frac{d+1}{d-1} Z_1\} (*) \\ = \zeta^T(k) \Lambda_5 Z_1 \Lambda_5^T \zeta(k), \end{aligned} \quad (31)$$

where

$$\begin{cases} Z_1 := \text{diag}\{Z_1, 3(\frac{d+1}{d-1})Z_1\}, \\ \Lambda_5 := \begin{bmatrix} E_2 H^T - E_4 & E_2 H^T + E_4 - \frac{2}{d_1+1} E_6 \end{bmatrix}. \end{cases} \quad (32)$$

Similarly, by virtue of Lemma 2.2, the third and fourth terms in the RHS of (29) can be lower-bounded respectively by

$$\begin{aligned} d \sum_{s=k-d(k)}^{k-d_1-1} \bar{\eta}^T(s) H^T Z_2 H \bar{\eta}(s) \\ \geq \frac{1}{\alpha(k)} \begin{bmatrix} x(k-d_1) - x(k-d(k)) \\ x(k-d_1) + x(k-d(k)) - \frac{2}{d(k)-d_1+1} \vartheta_2(k) \end{bmatrix}^T \\ \times \text{diag}\{Z_2, 3Z_2\} (*), \\ d \sum_{s=k-d_2}^{k-d(k)-1} \bar{\eta}^T(s) H^T Z_2 H \bar{\eta}(s) \\ \geq \frac{1}{\beta(k)} \begin{bmatrix} x(k-d(k)) - x(k-d_2) \\ x(k-d(k)) + x(k-d_2) - \frac{2}{d_2-d(k)+1} \vartheta_3(k) \end{bmatrix}^T \\ \times \text{diag}\{Z_2, 3Z_2\} (*), \end{aligned} \quad (33)$$

where $\alpha(k) := \frac{d(k)-d_1}{d}$ and $\beta(k) := 1 - \alpha(k)$.

It is known that $d_1 \leq d(k) \leq d_2$, and thus $\alpha(k)$ satisfies $0 < \alpha(k) < 1$. Then by Lemma A1 (referring to the Appendix), the subsequent inequality holds for a suitable matrix $X \in \mathbb{R}^{2n_x \times 2n_x}$,

$$\begin{aligned} d \sum_{s=k-d(k)}^{k-d_1-1} \bar{\eta}^T(s) H^T Z_2 H \bar{\eta}(s) \\ + d \sum_{s=k-d_2}^{k-d(k)-1} \bar{\eta}^T(s) H^T Z_2 H \bar{\eta}(s) \\ \geq \begin{bmatrix} x(k-d_1) - x(k-d(k)) \\ x(k-d_1) + x(k-d(k)) - \frac{2}{d(k)-d_1+1} \vartheta_2(k) \\ x(k-d(k)) - x(k-d_2) \\ x(k-d(k)) + x(k-d_2) - \frac{2}{d_2-d(k)+1} \vartheta_3(k) \end{bmatrix}^T \\ \times \begin{bmatrix} \text{diag}\{Z_2, 3Z_2\} & * \\ X^T & \text{diag}\{Z_2, 3Z_2\} \end{bmatrix} (*) \\ = \zeta^T(k) \Lambda_6(d(k)) Z_2 \Lambda_6^T(d(k)) \zeta(k), \end{aligned} \quad (35)$$

where

$$\begin{cases} Z_2 := \begin{bmatrix} \text{diag}\{Z_2, 3Z_2\} & * \\ X^T & \text{diag}\{Z_2, 3Z_2\} \end{bmatrix}, \\ \Lambda_6(d(k)) := \begin{bmatrix} E_4 - E_3 & E_4 + E_3 - \frac{2}{d(k)-d_1+1} E_7 \\ E_3 - E_5 & E_3 + E_5 - \frac{2}{d_2-d(k)+1} E_8 \end{bmatrix}. \end{cases} \quad (36)$$

Furthermore, by considering equation (14), for any appropriately dimensioned matrix \mathcal{G}_i , one has

$$\begin{aligned} 2\zeta^T(k) \mathcal{G}_i [-\bar{x}(k+1) + \bar{\mathcal{A}}_i \bar{x}(k) + \bar{\mathcal{A}}_{di} x(k-d(k)) \\ + \bar{\mathcal{B}}_i w(k) + \bar{\mathbf{a}}_i] = 0. \end{aligned} \quad (37)$$

Now, adding the terms on the left-hand side (LHS) of equation (37) to the LHS of (24), together with (27), (28), (31), and (35), we have

$$\begin{aligned} \text{LHS}(24) &\leq \zeta^T(k) (\text{Sym}\{\mathcal{G}_i \mathcal{A}_i\} + \Lambda_1 P_j(\mu^+) \Lambda_1^T \\ &\quad - \Lambda_2 P_i(\mu) \Lambda_2^T + \Lambda_3 \mathcal{R} \Lambda_3^T + \Lambda_4 Z_0 \Lambda_4^T \\ &\quad - \Lambda_5 Z_1 \Lambda_5^T - \Lambda_6(d(k)) Z_2 \Lambda_6^T(d(k)) \\ &\quad + \mathcal{L}_i^T \mathcal{L}_i - \gamma^2 E_9 E_9^T) \zeta(k) \end{aligned} \quad (38)$$

where

$$\begin{cases} Z_0 := \begin{bmatrix} H^T (d_1^2 Z_1 + d^2 Z_2) H & * \\ -H^T (d_1^2 Z_1 + d^2 Z_2) H & H^T (d_1^2 Z_1 + d^2 Z_2) H \end{bmatrix}, \\ \Lambda_4 := \begin{bmatrix} E_1 & E_2 \end{bmatrix}, \end{cases}$$

$$\begin{cases} \mathcal{A}_i := \begin{bmatrix} -\mathbf{I} & \bar{\mathcal{A}}_i & \bar{\mathcal{A}}_{di} & \mathbf{0}_{2n_x \times 5n_x} & \bar{\mathcal{B}}_i & \bar{\mathbf{a}}_i \\ \mathbf{0}_{n_z \times 2n_x} & \bar{\mathcal{L}}_i & \bar{\mathcal{L}}_{di} & \mathbf{0}_{n_z \times 5n_x} & \bar{\mathcal{D}}_i & \mathbf{0}_{n_z \times 1} \end{bmatrix}, \\ \mathcal{L}_i := \begin{bmatrix} \mathbf{0}_{n_z \times 2n_x} & \bar{\mathcal{L}}_i & \bar{\mathcal{L}}_{di} & \mathbf{0}_{n_z \times 5n_x} & \bar{\mathcal{D}}_i & \mathbf{0}_{n_z \times 1} \end{bmatrix}, \end{cases} \quad (39)$$

and E_9 is defined in (26).

Then, on the basis of the structural partitioning information in (11) and utilizing the S-procedure [45], one obtains the subsequent inequality, which implies (38) in the context of $\lambda_i < 0, i \in \mathcal{I}_1$,

$$\text{LHS}(38) + \lambda_i \begin{bmatrix} x(k) \\ 1 \end{bmatrix}^T \begin{bmatrix} Q_i^T Q_i & Q_i^T q_i \\ q_i^T Q_i & q_i^T q_i - 1 \end{bmatrix} \begin{bmatrix} x(k) \\ 1 \end{bmatrix} < 0. \quad (40)$$

It is obvious that the condition (18) is followed by (40), which means that the FES (14) is stable with an \mathcal{H}_∞ performance according to the Lyapunov stability theory. This completes the proof. ■

Remark 3.1. In view of the piecewise-fuzzy LKF (21), together with the new summation inequality (Lemmas 2.1 and 2.2), improved reciprocally convex inequality (Lemma A1) and S-procedure, a new bounded real lemma is provided in Theorem 3.1. It is noted that with the introduction of slack matrix variables \mathcal{G}_i , the Lyapunov matrices in Theorem 3.1 have been decoupled from the system matrices in any terms. It is also worth mentioning that this separation characteristic facilitates us to employ a piecewise-fuzzy Lyapunov functional to tackle the analysis and design problems of fuzzy systems. From this perspective, it is anticipated that the resulting condition shall be less conservative owing to the enlarged freedom from those piecewise-fuzzy Lyapunov matrices and additional free variables.

It is noted that the condition (18) involves the time-varying and nonlinear quadratic term $-\Lambda_6(d(k))\mathcal{Z}_2\Lambda_6^T(d(k))$, which is nonconvex and inconvenient for numerically tractable analysis and synthesis. With the Projection lemma [43], a more numerically tractable condition for performance analysis will be given in the next theorem.

Theorem 3.2. The FES in (14) is stable with an \mathcal{H}_∞ performance γ , if there exist matrices $P_i(\mu) \in \mathbb{S}^{4n_x}$, $\{R_1, R_2, Z_1, Z_2\} \in \mathbb{S}^{n_x}$, $\bar{G}_{1i} \in \mathbb{R}^{(10n_x+n_w) \times 2n_x}$, $X \in \mathbb{R}^{2n_x \times 2n_x}$, $F_0 \in \mathbb{R}^{4n_x \times (10n_x+n_w)}$, $i \in \mathcal{I}$, and $\bar{G}_{2i} \in \mathbb{R}^{1 \times 2n_x}$, $F_1 \in \mathbb{R}^{4n_x \times 1}$, and scalar $\lambda_i < 0, i \in \mathcal{I}_1$, guaranteeing that the subsequent matrix inequalities hold,

$$\begin{bmatrix} \bar{\Xi}_{ij}^{(\ell)}(\mu, \mu^+) & * \\ F & -Z_2 \end{bmatrix} < 0, \quad i \in \mathcal{I}, \quad (i, j) \in \Omega, \quad \ell = 1, 2, \quad (41)$$

where

$$\begin{cases} \bar{\Xi}_{ij}^{(\ell)}(\mu, \mu^+) := \text{Sym}\{\mathcal{G}_i\mathcal{A}_i + \Lambda_6^{(\ell)}F\} + \Lambda_1 P_j(\mu^+) \Lambda_1^T \\ \quad - \Lambda_2 P_i(\mu) \Lambda_2^T + \Lambda_3 \mathcal{R} \Lambda_3^T + \Lambda_4 \mathcal{Z}_0 \Lambda_4^T \\ \quad - \Lambda_5 \mathcal{Z}_1 \Lambda_5^T + \bar{\lambda}_i \Lambda_7 \mathcal{Q}_i \Lambda_7^T \\ \quad + \mathcal{L}_i^T \mathcal{L}_i - \gamma^2 E_9 E_9^T, \\ \Lambda_6^{(1)} := \begin{bmatrix} E_4 - E_3 & E_4 + E_3 - 2E_7 \\ E_3 - E_5 & E_3 + E_5 - \frac{2}{d+1}E_8 \end{bmatrix}, \\ \Lambda_6^{(2)} := \begin{bmatrix} E_4 - E_3 & E_4 + E_3 - \frac{2}{d+1}E_7 \\ E_3 - E_5 & E_3 + E_5 - 2E_8 \end{bmatrix}, \\ F := \begin{cases} F_0, & \text{if } i \in \mathcal{I}_0, \\ [F_0 \quad F_1], & \text{if } i \in \mathcal{I}_1, \end{cases} \end{cases} \quad (42)$$

with the other notations defined the same as in (19).

Proof. Based on Theorem 3.1, the FES (14) is stable with an \mathcal{H}_∞ performance γ if the condition (18) holds.

Firstly, rewrite (18) as

$$\Gamma(d(k))\Theta_{ij}(\mu, \mu^+)\Gamma^T(d(k)) < 0, \quad (43)$$

where

$$\begin{cases} \Gamma(d(k)) := [\mathbf{I}_{(8n_x+2n_u+n_w+1)} & \Lambda_6(d(k))] , \\ \Theta_{ij}(\mu, \mu^+) := \text{diag}\{\Xi_{ij}(\mu, \mu^+), -Z_2\}, \\ \Xi_{ij}(\mu, \mu^+) := \text{Sym}\{\mathcal{G}_i\mathcal{A}_i\} + \Lambda_1 P_j(\mu^+) \Lambda_1^T - \Lambda_2 P_i(\mu) \Lambda_2^T \\ \quad + \Lambda_3 \mathcal{R} \Lambda_3^T + \Lambda_4 \mathcal{Z}_0 \Lambda_4^T - \Lambda_5 \mathcal{Z}_1 \Lambda_5^T \\ \quad + \bar{\lambda}_i \Lambda_7 \mathcal{Q}_i \Lambda_7^T + \mathcal{L}_i^T \mathcal{L}_i - \gamma^2 E_9 E_9^T. \end{cases} \quad (44)$$

Applying Projection lemma [43] to (43), gives the following inequality,

$$\Theta_{ij}(\mu, \mu^+) + \text{Sym}\{\Gamma_\perp(d(k))\check{F}\} < 0. \quad (45)$$

Similar to [29], we specify $\check{F} := [F \quad \mathbf{0}_{4n_x \times 4n_x}]$, where $F \in \mathbb{R}^{4n_x \times (10n_x+n_w+1)}$. Furthermore, we can rewrite the condition (46) as,

$$\begin{bmatrix} \check{\Xi}_{ij}(\mu, \mu^+, d(k)) & * \\ F & -Z_2 \end{bmatrix} < 0, \quad i \in \mathcal{I}_1, \quad (46)$$

where

$$\begin{aligned} \check{\Xi}_{ij}(\mu, \mu^+, d(k)) &:= \text{Sym}\{\mathcal{G}_i\mathcal{A}_i + \Lambda_6(d(k))F\} \\ &\quad + \Lambda_1 P_j(\mu^+) \Lambda_1^T - \Lambda_2 P_i(\mu) \Lambda_2^T \\ &\quad + \Lambda_3 \mathcal{R} \Lambda_3^T + \Lambda_4 \mathcal{Z}_0 \Lambda_4^T - \Lambda_5 \mathcal{Z}_1 \Lambda_5^T \\ &\quad + \bar{\lambda}_i \Lambda_7 \mathcal{Q}_i \Lambda_7^T + \mathcal{L}_i^T \mathcal{L}_i - \gamma^2 E_9 E_9^T. \end{aligned} \quad (47)$$

Since the condition in (46) is affine with respect to the time-varying delay $d(k)$, with the upper bound d_2 and lower bound d_1 of delay $d(k)$ we have that (46) leads to (41) directly. Similarly, it can also be shown that (41) implies (18) in the context of $i \in \mathcal{I}_0$.

Therefore, one can immediately conclude that the FES (14) is stable with an \mathcal{H}_∞ performance γ if (41) holds. The proof is completed. ■

In the following subsection, on the basis of the new delay-dependent \mathcal{H}_∞ performance analysis condition proposed in Theorem 3.2, we shall develop the filter synthesis procedure.

B. Delay-Dependent Piecewise-Affine \mathcal{H}_∞ Filter Design

Theorem 3.3. Consider the fuzzy-affine system (1). If there exist matrices $P_{is} \in \mathbb{S}^{4n_x}$, $\{R_1, R_2, Z_1, Z_2\} \in \mathbb{S}^{n_x}$, $X \in \mathbb{R}^{2n_x \times 2n_x}$, $\{G_{1i(1)}, G_{1i(2)}, G_{1i(3)}, G_{2i(1)}, G_{2i(2)}, \hat{A}_{fi}\} \in \mathbb{R}^{n_x \times n_x}$, $G_{3i(1)} \in \mathbb{R}^{(6n_x+n_w) \times n_x}$, $B_{fi} \in \mathbb{R}^{n_x \times n_y}$, $C_{fi} \in \mathbb{R}^{n_z \times n_x}$, $D_{fi} \in \mathbb{R}^{n_z \times n_y}$, $F_0 \in \mathbb{R}^{4n_x \times (10n_x+n_w)}$, $i \in \mathcal{I}$, and $G_{4i(1)} \in \mathbb{R}^{1 \times n_x}$, $\hat{a}_{fi} \in \mathbb{R}^{n_x \times 1}$, $F_1 \in \mathbb{R}^{4n_x \times 1}$, and scalar $\lambda_i < 0, i \in \mathcal{I}_1$, guaranteeing that the subsequent LMIs hold,

$$\begin{bmatrix} \bar{\Xi}_{ijsv}^{(\ell)} & * & * \\ F & -Z_2 & * \\ \bar{\mathcal{L}}_{is} & \mathbf{0} & -\mathbf{I} \end{bmatrix} < 0, \quad (48)$$

$i \in \mathcal{I}, \quad (i, j) \in \Omega, \quad s, v \in \mathcal{S}(i), \quad \ell = 1, 2,$

where

$$\left\{ \begin{array}{l} \hat{\Xi}_{ijsv}^{(\ell)} := \text{Sym}\{G_i \mathbb{A}_s + \Pi \mathbb{A}_{fis} + \Lambda_6^{(\ell)} F\} + \Lambda_1 P_{jv} \Lambda_1^T \\ \quad - \Lambda_2 P_{is} \Lambda_2^T + \Lambda_3 \mathcal{R} \Lambda_3^T + \Lambda_4 \mathcal{Z}_0 \Lambda_4^T - \Lambda_5 \mathcal{Z}_1 \Lambda_5^T \\ \quad + \bar{\lambda}_i \Lambda_7 \mathcal{Q}_i \Lambda_7^T - \gamma^2 E_9 E_9^T, \\ \mathbb{G}_i := \begin{bmatrix} G_{1i(1)}^T & G_{1i(2)}^T & G_{2i(1)}^T & G_{2i(2)}^T \\ & & G_{3i(1)}^T & \\ & & & G_{4i(1)}^T \end{bmatrix}^T, \\ \mathbb{A}_s := \begin{bmatrix} -\mathbf{I} & \mathbf{0}_{n_x \times n_x} & A_s & \mathbf{0}_{n_x \times n_x} \\ & A_{ds} & \mathbf{0}_{n_x \times 5n_x} & B_s \end{bmatrix}, \\ \Pi := \begin{bmatrix} \mathbf{I} & \mathbf{I} & \delta_1 \mathbf{I} & \delta_2 \mathbf{I} & \mathbf{0}_{n_x \times (6n_x + n_w)} \end{bmatrix}^T, \\ \mathbb{A}_{fis} := \begin{bmatrix} \mathbf{0}_{n_x \times n_x} & -G_{1i(3)} & \hat{B}_{fi} C_s & \hat{A}_{fi} \\ & \hat{B}_{fi} C_{ds} & \mathbf{0}_{n_x \times 5n_x} & \hat{B}_{fi} D_s \end{bmatrix}, \\ \hat{\mathcal{L}}_{is} := \begin{bmatrix} \mathbf{0}_{n_z \times 2n_x} & L_s - D_{fi} C_s & -C_{fi} \\ L_{ds} - D_{fi} C_{ds} & \mathbf{0}_{n_z \times 5n_x} & -D_{fi} D_s \end{bmatrix}, \\ \text{if } i \in \mathcal{J}_0, \\ \mathbb{G}_i := \begin{bmatrix} G_{1i(1)}^T & G_{1i(2)}^T & G_{2i(1)}^T & G_{2i(2)}^T \\ & & G_{3i(1)}^T & G_{4i(1)}^T \end{bmatrix}^T, \\ \mathbb{A}_s := \begin{bmatrix} -\mathbf{I} & \mathbf{0}_{n_x \times n_x} & A_s & \mathbf{0}_{n_x \times n_x} & A_{ds} \\ & & \mathbf{0}_{n_x \times 5n_x} & B_s & a_s \end{bmatrix}, \\ \Pi := \begin{bmatrix} \mathbf{I} & \mathbf{I} & \delta_1 \mathbf{I} & \delta_2 \mathbf{I} & \mathbf{0}_{n_x \times (6n_x + n_w + 1)} \end{bmatrix}^T, \\ \mathbb{A}_{fis} := \begin{bmatrix} \mathbf{0}_{n_x \times n_x} & -G_{1i(3)} & \hat{B}_{fi} C_s & \hat{A}_{fi} \\ & \hat{B}_{fi} C_{ds} & \mathbf{0}_{n_x \times 5n_x} & \hat{B}_{fi} D_s & \hat{a}_{fi} \end{bmatrix}, \\ \hat{\mathcal{L}}_{is} := \begin{bmatrix} \mathbf{0}_{n_z \times 2n_x} & L_s - D_{fi} C_s & -C_{fi} \\ L_{ds} - D_{fi} C_{ds} & \mathbf{0}_{n_z \times 5n_x} & -D_{fi} D_s & \mathbf{0}_{n_z \times 1} \end{bmatrix}, \\ \text{if } i \in \mathcal{J}_1, \end{array} \right\}, \quad (49)$$

with the other notations defined the same as in (19) and (42), respectively, then the FES in (14) is stable with \mathcal{H}_∞ performance γ . Specifically, the admissible filter gains can be constructed as,

$$\begin{aligned} A_{fi} &= G_{1i(3)}^{-1} \hat{A}_{fi}, \quad B_{fi} = G_{1i(3)}^{-1} \hat{B}_{fi}, \quad C_{fi} = C_{fi}, \\ D_{fi} &= D_{fi}, \quad i \in \mathcal{J}, \quad a_{fi} = G_{1i(3)}^{-1} \hat{a}_{fi}, \quad i \in \mathcal{J}_1. \end{aligned} \quad (50)$$

Proof. By Theorem 3.2, the FES (14) is stable with an \mathcal{H}_∞ performance γ if the condition (41) holds. On the basis of (7), the LHS of inequality (41) can be easily re-organized as

$$\sum_{s \in \mathcal{S}(i)} \sum_{v \in \mathcal{S}(i)} \mu_s \mu_v \begin{bmatrix} \hat{\Xi}_{ijsv}^{(\ell)} & * \\ F & -\mathcal{Z}_2 \end{bmatrix} < 0, \quad i \in \mathcal{J}_1, \quad (51)$$

where

$$\left\{ \begin{array}{l} \hat{\Xi}_{ijsv}^{(\ell)} := \text{Sym}\{\mathcal{G}_i \hat{\mathcal{A}}_{is} + \Lambda_6^{(\ell)} F\} + \Lambda_1 P_{jv} \Lambda_1^T - \Lambda_2 P_{is} \Lambda_2^T \\ \quad + \Lambda_3 \mathcal{R} \Lambda_3^T + \Lambda_4 \mathcal{Z}_0 \Lambda_4^T - \Lambda_5 \mathcal{Z}_1 \Lambda_5^T \\ \quad + \bar{\lambda}_i \Lambda_7 \mathcal{Q}_i \Lambda_7^T + \hat{\mathcal{L}}_{is}^T \hat{\mathcal{L}}_{is} - \gamma^2 E_9 E_9^T, \\ \hat{\mathcal{A}}_{is} := \begin{bmatrix} -\mathbf{I} & \begin{bmatrix} A_s & \mathbf{0} \\ B_{fi} C_s & A_{fi} \end{bmatrix} \\ & \mathbf{0}_{2n_x \times 5n_x} \end{bmatrix} \begin{bmatrix} A_{ds} \\ B_{fi} C_{ds} \\ B_{fi} D_s \end{bmatrix} \begin{bmatrix} a_s \\ a_{fi} \end{bmatrix}, \\ \hat{\mathcal{L}}_{is} := \begin{bmatrix} \mathbf{0}_{n_z \times 2n_x} & L_s - D_{fi} C_s & -C_{fi} \\ L_{ds} - D_{fi} C_{ds} & \mathbf{0}_{n_z \times 5n_x} & -D_{fi} D_s & \mathbf{0}_{n_z \times 1} \end{bmatrix}. \end{array} \right. \quad (52)$$

Considering the intrinsic nonnegative property of the fuzzy-basis functions, by the Schur complement, the following

inequality implies (52):

$$\begin{bmatrix} \hat{\Xi}_{ijsv}^{(\ell)} & * & * \\ F & -\mathcal{Z}_2 & * \\ \hat{\mathcal{L}}_{is} & \mathbf{0} & -\mathbf{I} \end{bmatrix} < 0, \quad i \in \mathcal{J}_1, \quad s, v \in \mathcal{S}(i), \quad (i, j) \in \Omega, \quad (53)$$

where

$$\begin{aligned} \hat{\Xi}_{ijsv}^{(\ell)} &:= \text{Sym}\{\mathcal{G}_i \hat{\mathcal{A}}_{is} + \Lambda_6^{(\ell)} F\} + \Lambda_1 P_{jv} \Lambda_1^T - \Lambda_2 P_{is} \Lambda_2^T \\ &\quad + \Lambda_3 \mathcal{R} \Lambda_3^T + \Lambda_4 \mathcal{Z}_0 \Lambda_4^T - \Lambda_5 \mathcal{Z}_1 \Lambda_5^T \\ &\quad + \bar{\lambda}_i \Lambda_7 \mathcal{Q}_i \Lambda_7^T - \gamma^2 E_9 E_9^T. \end{aligned} \quad (54)$$

For the numerical tractability of the filter synthesis conditions, we first prescribe the slack variables as,

$$\left\{ \begin{array}{l} \mathcal{G}_i := \begin{bmatrix} G_{1i}^T & \epsilon_1 G_{1i}^T & \mathbf{0}_{2n_x \times (6n_x + n_w + 1)} \end{bmatrix}^T, \\ G_{1i} := \begin{bmatrix} G_{1i(1)} & G_{1i(3)} \\ G_{1i(2)} & G_{1i(4)} \end{bmatrix}, \end{array} \right. \quad (55)$$

where ϵ_1 is a scalar parameter, and $\{G_{1i(1)}, G_{1i(2)}, G_{1i(3)}, G_{1i(4)}\} \in \mathbb{R}^{n_x \times n_x}$. Then, to linearize the matrix inequality, similar to [29], making a congruent transformation to

$$\begin{bmatrix} G_{1i(1)} + G_{1i(1)}^T & * \\ G_{1i(3)}^T + G_{1i(2)} & G_{1i(4)} + G_{1i(4)}^T \end{bmatrix} \quad (56)$$

by $\text{diag}\{\mathbf{I}_{n_x}, G_{1i(3)}^{-1} G_{1i(4)}^{-1}\}$ yields

$$\begin{aligned} &\begin{bmatrix} G_{1i(1)} + G_{1i(1)}^T & * \\ G_{1i(3)} G_{1i(4)}^{-1} G_{1i(3)}^T + G_{1i(3)} G_{1i(4)}^{-1} G_{1i(2)} & * \\ G_{1i(3)} G_{1i(4)}^{-1} G_{1i(3)}^T + G_{1i(3)} G_{1i(4)}^{-1} G_{1i(2)} & * \\ G_{1i(3)} G_{1i(4)}^{-1} G_{1i(3)}^T + G_{1i(3)} G_{1i(4)}^{-1} G_{1i(2)} & * \end{bmatrix} \\ &:= \begin{bmatrix} G_{1i(1)} + G_{1i(1)}^T & * \\ \bar{G}_{1i(3)}^T + \bar{G}_{1i(2)} & \bar{G}_{1i(3)} + \bar{G}_{1i(3)}^T \end{bmatrix}. \end{aligned} \quad (57)$$

Therefore, it is reasonable to re-specify the matrix G_{1i} in (55) as,

$$G_{1i} := \begin{bmatrix} G_{1i(1)} & G_{1i(3)} \\ G_{1i(2)} & G_{1i(3)} \end{bmatrix}, \quad i \in \mathcal{J}_1. \quad (58)$$

Thanks to the special structures of G_{1i} , the matrix variable $G_{1i(3)}$ can be incorporated into the variables A_{fi} , B_{fi} and a_{fi} by defining

$$\hat{A}_{fi} := G_{1i(3)} A_{fi}, \quad \hat{B}_{fi} := G_{1i(3)} B_{fi}, \quad \hat{a}_{fi} := G_{1i(3)} a_{fi}. \quad (59)$$

Besides, by inspection of the inner structure of system matrices in (15), we can see that the filter gains do not appear in the first row of the system matrices $\hat{\mathcal{A}}_i$, $\bar{\mathcal{A}}_{di}$, \bar{B}_i and \bar{a}_i . For further conservatism reduction of the design procedure, we can thus prescribe the slack variables as

$$\mathcal{G}_i := \begin{bmatrix} G_{1i}^T & G_{2i}^T & G_{3i}^T & G_{4i}^T \end{bmatrix}^T, \quad i \in \mathcal{J}_1, \quad (60)$$

where

$$\left\{ \begin{array}{l} G_{1i} := \begin{bmatrix} G_{1i(1)} & G_{1i(3)} \\ G_{1i(2)} & G_{1i(3)} \end{bmatrix}, \\ G_{2i} := \begin{bmatrix} G_{2i(1)} & \delta_1 G_{1i(3)} \\ G_{2i(2)} & \delta_2 G_{1i(3)} \end{bmatrix}, \\ G_{3i} := \begin{bmatrix} G_{3i(1)} & \mathbf{0} \end{bmatrix}, \\ G_{4i} := \begin{bmatrix} G_{4i(1)} & \mathbf{0} \end{bmatrix}, \end{array} \right. \quad (61)$$

with $\{G_{1i(1)}, G_{1i(2)}, G_{1i(3)}, G_{2i(1)}, G_{2i(2)}\} \in \mathbb{R}^{n_x \times n_x}$, $G_{3i(1)} \in \mathbb{R}^{(6n_x+n_w) \times n_x}$, $G_{4i(1)} \in \mathbb{R}^{1 \times n_x}$, and scalar parameters δ_1 and δ_2 .

Then, by substitution of matrix \mathcal{G}_i defined in (60) into (41) and the application of Schur complement, one readily obtains (48) with consideration of (59).

In addition, the conditions in (48) result in $-G_{1i(3)} - G_{1i(3)}^T < 0$, which intuitively indicates that $G_{1i(3)}$ is invertible. Henceforth, the filter parameters can be easily derived from (50). The proof is completed. ■

Remark 3.2. Notice that by the state-augmentation (37), the linearization manner and algebraic operations in Theorem 3.3 developed above claim a much more straightforward circumstance than most conventional approaches [31], [39], where some constraints are enforced on the Lyapunov function variables P_{is} , $i \in \mathcal{I}$, $s \in \mathcal{S}(i)$. Especially, this linearization technique facilitates one to select more slack variables in the matrix \mathcal{G}_i , i.e., every variable in the first block-column of the matrix \mathcal{G}_i hosts explicitly freedom and suffers from no structural constraints, which will result in the conservatism reduction of the solutions.

For a more intuitive understanding of the advantages of the above developed synthesis method, in the following, we also present another filter synthesis result based on a conventional decoupling inequality: $-\Phi^T P^{-1} \Phi \leq P - \Phi - \Phi^T$ with $P > 0$, but without using state-augmentation equation (37). For fair comparison purposes, the same piecewise-affine LKF in (21)-(23) is employed. The results are immediately summarized in the subsequent corollaries.

Corollary 3.1. The FES in (14) is stable with an \mathcal{H}_∞ performance γ , if there exist matrices $P_i(\mu) \in \mathbb{S}^{4n_x}$, $\{R_1, R_2, Z_1, Z_2\} \in \mathbb{S}^{n_x}$, $X \in \mathbb{R}^{2n_x \times 2n_x}$, $i \in \mathcal{I}$, and scalar $\lambda_i < 0$, $i \in \mathcal{I}_1$, guaranteeing that the subsequent matrix inequalities hold,

$$\begin{aligned} & \bar{\Lambda}_{1i} P_j(\mu^+) \bar{\Lambda}_{1i}^T - \bar{\Lambda}_{2i} P_i(\mu) \bar{\Lambda}_{2i}^T + \bar{\Lambda}_{3i} \mathcal{R} \bar{\Lambda}_{3i}^T + \bar{\Lambda}_{4i} \mathcal{Z}_0 \bar{\Lambda}_{4i}^T \\ & - \bar{\Lambda}_{5i} \mathcal{Z}_1 \bar{\Lambda}_{5i}^T - \bar{\Lambda}_6(d(k)) \mathcal{Z}_2 \bar{\Lambda}_6^T(d(k)) + \bar{\lambda}_i \bar{\Lambda}_7 \mathcal{Q}_i \bar{\Lambda}_7^T \\ & + \mathcal{L}_i^T \mathcal{L}_i - \gamma^2 \bar{E}_8 \bar{E}_8^T < 0, i \in \mathcal{I}, (i, j) \in \Omega, \end{aligned} \quad (62)$$

where

$$\left\{ \begin{aligned} \bar{\Lambda}_1 &:= [\bar{\mathcal{A}}_i \quad \bar{E}_5 - \bar{E}_3 \quad \bar{E}_6 + \bar{E}_7 - \bar{E}_2 - \bar{E}_4], \\ \bar{\Lambda}_2 &:= [\bar{E}_1 \quad \bar{E}_5 - \bar{E}_1 H^T \quad \bar{E}_6 + \bar{E}_7 - \bar{E}_2 - \bar{E}_3], \\ \bar{\Lambda}_3 &:= [\bar{E}_1 \quad \bar{E}_3 \quad \bar{E}_4], \\ \bar{\Lambda}_{4i} &:= [\bar{\mathcal{A}}_i \quad \bar{E}_1], \\ \bar{\Lambda}_5 &:= [\bar{E}_1 H^T - \bar{E}_3 \quad \bar{E}_1 H^T + \bar{E}_3 - \frac{2}{d_1+1} \bar{E}_5], \\ \bar{\Lambda}_6(d(k)) &:= [\bar{E}_3 - \bar{E}_2 \quad \bar{E}_3 + \bar{E}_2 - \frac{2}{d(k)-d_1+1} \bar{E}_6 \\ & \quad \bar{E}_2 - \bar{E}_4 \quad \bar{E}_2 + \bar{E}_4 - \frac{2}{d_2-d(k)+1} \bar{E}_7], \\ \bar{\mathcal{A}}_i &:= [\bar{\mathcal{A}}_i \quad \bar{\mathcal{A}}_{di} \quad \mathbf{0}_{2n_x \times 5n_x} \quad \bar{\mathcal{B}}_i], \\ \bar{\mathcal{L}}_i &:= [\bar{\mathcal{L}}_i \quad \bar{\mathcal{L}}_{di} \quad \mathbf{0}_{n_z \times 5n_x} \quad \bar{\mathcal{D}}_i], \\ \bar{E}_\iota &:= [\underbrace{\mathbf{0} \cdots \mathbf{0}}_{\iota-1} \quad \mathbf{I}_{\bar{\sigma}(\iota)} \quad \underbrace{\mathbf{0} \cdots \mathbf{0}}_{8-\iota}]^T, \end{aligned} \right\}, \text{ if } i \in \mathcal{I}_0,$$

$$\bar{E}_\iota := [\underbrace{\mathbf{0} \cdots \mathbf{0}}_{\iota-1} \quad \mathbf{I}_{\bar{\sigma}(\iota)} \quad \underbrace{\mathbf{0} \cdots \mathbf{0}}_{8-\iota}]^T, \text{ if } i \in \mathcal{I}_1,$$

$$\left\{ \begin{aligned} \bar{\mathcal{A}}_i &:= [\bar{\mathcal{A}}_i \quad \bar{\mathcal{A}}_{di} \quad \mathbf{0}_{2n_x \times 5n_x} \quad \bar{\mathcal{B}}_i \quad \bar{\mathbf{a}}_i], \\ \bar{\mathcal{L}}_i &:= [\bar{\mathcal{L}}_i \quad \bar{\mathcal{L}}_{di} \quad \mathbf{0}_{n_z \times 5n_x} \quad \bar{\mathcal{D}}_i \quad \mathbf{0}_{n_z \times 1}], \\ \bar{E}_\iota &:= [\underbrace{\mathbf{0} \cdots \mathbf{0}}_{\iota-1} \quad \mathbf{I}_{\bar{\sigma}(\iota)} \quad \underbrace{\mathbf{0} \cdots \mathbf{0}}_{9-\iota}]^T, \\ &\in \mathbb{R}^{(8n_x+n_w+1) \times \bar{\sigma}(\iota)}, \iota = 1, \dots, 9, \\ \bar{\Lambda}_7 &:= [\bar{E}_1 \quad \bar{E}_9], \end{aligned} \right\}, \quad \text{if } i \in \mathcal{I}_1, \quad (63)$$

with

$$\bar{\sigma}(\iota) := \begin{cases} 2n_x, & \iota = 1, \\ n_x, & \iota = 2, \dots, 7, \\ n_w, & \iota = 8, \\ 1, & \iota = 9, \end{cases} \quad (64)$$

and the other notations are defined in (19).

Proof. Based on the piecewise-fuzzy LKF utilized in (21)-(23) and by taking the similar derivation procedures as in the proof of Theorem 3.1, one obtains the conditions (27), (28), (31) and (35) in the same manner. Bearing in mind the equations in (14) and by expanding the state variable $\bar{x}(k+1)$ in (27) and (31), the upper bound of LHS(24) can be reformulated as,

$$\begin{aligned} \text{LHS}(24) &\leq \bar{\zeta}^T(k) (\bar{\Lambda}_{1i} P_j(\mu^+) \bar{\Lambda}_{1i}^T - \bar{\Lambda}_{2i} P_i(\mu) \bar{\Lambda}_{2i}^T \\ & \quad + \bar{\Lambda}_{3i} \mathcal{R} \bar{\Lambda}_{3i}^T + \bar{\Lambda}_{4i} \mathcal{Z}_0 \bar{\Lambda}_{4i}^T - \bar{\Lambda}_{5i} \mathcal{Z}_1 \bar{\Lambda}_{5i}^T \\ & \quad - \bar{\Lambda}_6(d(k)) \mathcal{Z}_2 \bar{\Lambda}_6^T(d(k)) + \mathcal{L}_i^T \mathcal{L}_i \\ & \quad - \gamma^2 \bar{E}_9 \bar{E}_9^T) \bar{\zeta}(k) \end{aligned} \quad (65)$$

where

$$\left\{ \begin{aligned} \bar{\zeta}(k) &:= [\bar{x}^T(k) \quad x^T(k-d(k)) \\ & \quad x^T(k-d_1) \quad x^T(k-d_2) \quad \vartheta_1^T(k) \\ & \quad \vartheta_2^T(k) \quad \vartheta_3^T(k) \quad w^T(k) \quad 1], \\ \vartheta_1(k) &:= \sum_{s=k-d_1}^k x(s), \\ \vartheta_2(k) &:= \sum_{s=k-d(k)}^{k-d_1} x(s), \\ \vartheta_3(k) &:= \sum_{s=k-d_2}^{k-d(k)} x(s), \end{aligned} \right. \quad (66)$$

and the other notations are defined in (63).

It can be easily checked that similar lines involving an S-procedure with $\lambda_i < 0$, $i \in \mathcal{I}_1$, lead to the condition (62). This completes the proof. ■

Similarly, we can formulate the subsequent \mathcal{H}_∞ performance analysis condition in a convex optimization framework.

Corollary 3.2. The FES in (14) is stable with an \mathcal{H}_∞ performance γ , if there exist matrices $P_i(\mu) \in \mathbb{S}^{4n_x}$, $\{R_1, R_2, Z_1, Z_2\} \in \mathbb{S}^{n_x}$, $X \in \mathbb{R}^{2n_x \times 2n_x}$, $\bar{F}_0 \in \mathbb{R}^{4n_x \times (8n_x+n_w)}$, $i \in \mathcal{I}$, and $\bar{F}_1 \in \mathbb{R}^{4n_x \times 1}$, and scalar $\lambda_i < 0$, $i \in \mathcal{I}_1$, guaranteeing that the subsequent matrix inequalities hold,

$$\left[\begin{array}{cc} \Psi_{ij}^{(\ell)}(\mu, \mu^+) & * \\ F & -Z_2 \end{array} \right] < 0, \quad i \in \mathcal{I}, (i, j) \in \Omega, \ell = 1, 2, \quad (67)$$

where

$$\left\{ \begin{array}{l} \Psi_{ij}^{(\ell)}(\mu, \mu^+) := \text{Sym}\{\Lambda_6^{(\ell)} \bar{F}\} + \bar{\Lambda}_{1i} P_j(\mu^+) \bar{\Lambda}_1^T \\ \quad - \bar{\Lambda}_2 P_i(\mu) \bar{\Lambda}_2^T + \bar{\Lambda}_3 \mathcal{R} \bar{\Lambda}_3^T + \bar{\Lambda}_4 \mathcal{Z}_0 \bar{\Lambda}_{4i}^T \\ \quad - \bar{\Lambda}_5 \mathcal{Z}_1 \bar{\Lambda}_5^T + \bar{\lambda}_i \bar{\Lambda}_7 \mathcal{Q}_i \bar{\Lambda}_7^T \\ \quad + \bar{\mathcal{L}}_i^T \bar{\mathcal{L}}_i - \gamma^2 \bar{E}_8 \bar{E}_8^T, \\ \bar{\Lambda}_6^{(1)} := \begin{bmatrix} \bar{E}_3 - \bar{E}_2 & \bar{E}_3 + \bar{E}_2 - 2\bar{E}_6 \\ \bar{E}_2 - \bar{E}_4 & \bar{E}_2 + \bar{E}_4 - \frac{2}{d+1} \bar{E}_7 \end{bmatrix}, \\ \bar{\Lambda}_6^{(2)} := \begin{bmatrix} \bar{E}_3 - \bar{E}_2 & \bar{E}_3 + \bar{E}_2 - \frac{2}{d+1} \bar{E}_6 \\ \bar{E}_2 - \bar{E}_4 & \bar{E}_2 + \bar{E}_4 - 2\bar{E}_7 \end{bmatrix}, \\ \bar{F} := \begin{cases} \bar{F}_0, & \text{if } i \in \mathcal{I}_0, \\ \begin{bmatrix} \bar{F}_0 & \bar{F}_1 \end{bmatrix}, & \text{if } i \in \mathcal{I}_1, \end{cases} \end{array} \right. \quad (68)$$

with the other notations defined the same as in (63).

Furthermore, by a traditional linearization technique, the filter design condition will be developed based on Corollary 3.2.

Corollary 3.3. Consider the fuzzy-affine system (1). If there exist matrices $\{P_{is(1)}, P_{is(3)}\} \in \mathbb{S}^{2n_x}$, $\{R_1, R_2, Z_1, Z_2\} \in \mathbb{S}^{n_x}$, $X \in \mathbb{R}^{2n_x \times 2n_x}$, $\{P_{is(21)}, P_{is(22)}, \tilde{G}_{is(1)}, \tilde{G}_{is(2)}, \tilde{G}_{is(3)}, \tilde{A}_{fi}\} \in \mathbb{R}^{n_x \times n_x}$, $\tilde{B}_{fi} \in \mathbb{R}^{n_x \times n_y}$, $C_{fi} \in \mathbb{R}^{n_z \times n_x}$, $D_{fi} \in \mathbb{R}^{n_z \times n_y}$, $F_0 \in \mathbb{R}^{4n_x \times (8n_x + n_w)}$, $i \in \mathcal{I}$, and $\tilde{a}_{fi} \in \mathbb{R}^{n_x \times 1}$, $\bar{F}_1 \in \mathbb{R}^{4n_x \times 1}$, and scalar $\lambda_i < 0$, $i \in \mathcal{I}_1$, guaranteeing that the subsequent LMIs hold,

$$\left[\begin{array}{cccc} P_{jv(1)} - \text{Sym}\{\tilde{G}_{jv}\} & * & * & * \\ \tilde{\mathbf{A}}_{ijsv}^T & \tilde{\Psi}_{ijsv}^{(\ell)} & * & * \\ \mathbf{0} & \bar{F} & -Z_2 & * \\ \mathbf{0} & \bar{\mathcal{L}}_{is} & \mathbf{0} & -\mathbf{I} \end{array} \right] < 0, \quad (69)$$

$i \in \mathcal{I}, (i, j) \in \Omega, s, v \in \mathcal{S}(i), \ell = 1, 2,$

$$P_{is} := \left[\begin{array}{cc} P_{is(1)} & * \\ \begin{bmatrix} P_{is(21)}^T & P_{is(22)}^T \\ \rho_1 \tilde{G}_3 & \rho_2 \tilde{G}_3 \end{bmatrix} & P_{is(3)} \end{array} \right] > 0, \quad (70)$$

$i \in \mathcal{I}, s \in \mathcal{S}(i),$

where

$$\left\{ \begin{array}{l} \tilde{\Psi}_{ijsv}^{(\ell)} := \text{Sym}\{\bar{\Lambda}_6^{(\ell)} \bar{F}\} + \bar{\Lambda}_{1isv} \tilde{P}_{jv} \bar{\Lambda}_{1isv}^T - \bar{\Lambda}_2 P_{is} \bar{\Lambda}_2^T \\ \quad + \bar{\Lambda}_3 \mathcal{R} \bar{\Lambda}_3^T + \bar{\Lambda}_{4s}^T \mathcal{Z}_0 \bar{\Lambda}_{4s} - \bar{\Lambda}_5 \mathcal{Z}_1 \bar{\Lambda}_5^T \\ \quad + \bar{\lambda}_i \bar{\Lambda}_7 \mathcal{Q}_i \bar{\Lambda}_7^T - \gamma^2 \bar{E}_8 \bar{E}_8^T, \\ \tilde{\Lambda}_{1isv} := \begin{bmatrix} \tilde{\mathbf{A}}_{ijsv}^T & \bar{E}_5 - \bar{E}_3 & \bar{E}_6 + \bar{E}_7 - \bar{E}_2 - \bar{E}_4 \end{bmatrix}, \\ \tilde{\mathbf{A}}_{ijsv} := \begin{bmatrix} \tilde{\mathbf{A}}_{ijsv} & \tilde{\mathbf{A}}_{dijsv} & \mathbf{0}_{2n_x \times 5n_x} & \tilde{\mathbf{B}}_{ijsv} \end{bmatrix}, \\ \tilde{\mathbf{A}}_{dijsv} := \begin{bmatrix} \tilde{\mathbf{A}}_{ijsv} & \tilde{\mathbf{A}}_{dijsv} & \mathbf{0}_{2n_x \times 5n_x} & \tilde{\mathbf{B}}_{ijsv} \end{bmatrix}, \\ \tilde{\Lambda}_{4s} := \begin{bmatrix} A_s - \mathbf{I} & \mathbf{0}_{n_x \times n_x} & A_{ds} & \mathbf{0}_{n_x \times 5n_x} & B_s \end{bmatrix}, \\ \tilde{\mathcal{L}}_{is} := \begin{bmatrix} L_s - D_{fi} C_s & -C_{fi} & L_{ds} - D_{fi} C_{ds} & \mathbf{0}_{n_z \times 5n_x} & -D_{fi} D_s \end{bmatrix}, \\ \quad \text{if } i \in \mathcal{I}_0, \\ \tilde{\mathbf{A}}_{ijsv} := \begin{bmatrix} \tilde{\mathbf{A}}_{ijsv} & \tilde{\mathbf{A}}_{dijsv} & \mathbf{0}_{2n_x \times 5n_x} & \tilde{\mathbf{B}}_{ijsv} \\ \tilde{\mathbf{A}}_{ijsv} & \tilde{\mathbf{A}}_{dijsv} & \mathbf{0}_{2n_x \times 5n_x} & \tilde{\mathbf{B}}_{ijsv} \\ \tilde{\mathbf{A}}_{ijsv} & \tilde{\mathbf{A}}_{dijsv} & \mathbf{0}_{2n_x \times 5n_x} & \tilde{\mathbf{B}}_{ijsv} \end{bmatrix}, \\ \tilde{\mathbf{A}}_{dijsv} := \begin{bmatrix} \tilde{\mathbf{A}}_{ijsv} & \tilde{\mathbf{A}}_{dijsv} & \mathbf{0}_{2n_x \times 5n_x} & \tilde{\mathbf{B}}_{ijsv} \\ \tilde{\mathbf{A}}_{ijsv} & \tilde{\mathbf{A}}_{dijsv} & \mathbf{0}_{2n_x \times 5n_x} & \tilde{\mathbf{B}}_{ijsv} \\ \tilde{\mathbf{A}}_{ijsv} & \tilde{\mathbf{A}}_{dijsv} & \mathbf{0}_{2n_x \times 5n_x} & \tilde{\mathbf{B}}_{ijsv} \end{bmatrix}, \\ \tilde{\Lambda}_{4s} := \begin{bmatrix} A_s - \mathbf{I} & \mathbf{0}_{n_x \times n_x} & A_{ds} & \mathbf{0}_{n_x \times 5n_x} & B_s \\ & & & B_s & a_s \end{bmatrix}, \\ \tilde{\mathcal{L}}_{is} := \begin{bmatrix} L_s - D_{fi} C_s & -C_{fi} & L_{ds} - D_{fi} C_{ds} & \mathbf{0}_{n_z \times 5n_x} & -D_{fi} D_s & \mathbf{0}_{n_z \times 1} \end{bmatrix}, \\ \quad \text{if } i \in \mathcal{I}_1, \end{array} \right.$$

$$\left\{ \begin{array}{l} \tilde{\mathbf{A}}_{ijsv} := \begin{bmatrix} \tilde{G}_{jv(1)} A_s + \tilde{B}_{fi} C_s & \tilde{A}_{fi} \\ \tilde{G}_{jv(2)} A_s + \tilde{B}_{fi} C_s & \tilde{A}_{fi} \end{bmatrix}, \\ \tilde{\mathbf{A}}_{dijsv} := \begin{bmatrix} \tilde{G}_{jv(1)} A_{ds} + \tilde{B}_{fi} C_{ds} \\ \tilde{G}_{jv(2)} A_{ds} + \tilde{B}_{fi} C_{ds} \end{bmatrix}, \\ \tilde{\mathbf{B}}_{ijsv} := \begin{bmatrix} \tilde{G}_{jv(1)} B_s + \tilde{B}_{fi} D_s \\ \tilde{G}_{jv(2)} B_s + \tilde{B}_{fi} D_s \end{bmatrix}, \\ \tilde{\mathbf{a}}_{ijsv} := \begin{bmatrix} \tilde{G}_{jv(1)} a_s + \tilde{a}_{fi} \\ \tilde{G}_{jv(2)} a_s + \tilde{a}_{fi} \end{bmatrix}, \\ \tilde{\mathbf{Q}}_{ijsv} := \begin{bmatrix} P_{jv(21)}^T A_s + \rho_1 \tilde{B}_{fi} C_s & \rho_1 \tilde{A}_{fi} \\ P_{jv(22)}^T A_s + \rho_2 \tilde{B}_{fi} C_s & \rho_2 \tilde{A}_{fi} \end{bmatrix}, \\ \tilde{\mathbf{Q}}_{dijsv} := \begin{bmatrix} P_{jv(21)}^T A_{ds} + \rho_1 \tilde{B}_{fi} C_{ds} \\ P_{jv(22)}^T A_{ds} + \rho_2 \tilde{B}_{fi} C_{ds} \end{bmatrix}, \\ \tilde{\mathbf{B}}_{ijsv} := \begin{bmatrix} P_{jv(21)}^T B_s + \rho_1 \tilde{B}_{fi} D_s \\ P_{jv(22)}^T B_s + \rho_2 \tilde{B}_{fi} D_s \end{bmatrix}, \\ \tilde{\mathbf{a}}_{ijsv} := \begin{bmatrix} P_{jv(21)}^T a_s + \rho_1 \tilde{a}_{fi} \\ P_{jv(22)}^T a_s + \rho_2 \tilde{a}_{fi} \end{bmatrix}, \\ \tilde{P}_{jv} := \text{diag}\{\mathbf{0}_{2n_x \times 2n_x}, P_{jv(3)}\}, \\ \tilde{\mathcal{Z}}_0 := d_1^2 Z_1 + d_2^2 Z_2, \\ \tilde{G}_{jv} := \begin{bmatrix} \tilde{G}_{jv(1)} & \tilde{G}_3 \\ \tilde{G}_{jv(2)} & \tilde{G}_3 \end{bmatrix}, \end{array} \right. \quad (71)$$

and the other notations defined the same as in (19) and (63), respectively, then the FES in (14) is stable with an \mathcal{H}_∞ performance γ . Specifically, the admissible filter can be constructed as,

$$\begin{aligned} A_{fi} &= G_{1i(3)}^{-1} \tilde{A}_{fi}, \quad B_{fi} = G_{1i(3)}^{-1} \tilde{B}_{fi}, \quad C_{fi} = C_{fi}, \\ D_{fi} &= D_{fi}, \quad i \in \mathcal{I}, \quad a_{fi} = G_{1i(3)}^{-1} \tilde{a}_{fi}, \quad i \in \mathcal{I}_1. \end{aligned} \quad (72)$$

Proof. Based on (7), the inequality (67) can be readily rewritten as

$$\sum_{s \in \mathcal{S}(i)} \sum_{v \in \mathcal{S}(i)} \mu_s \mu_v \left[\begin{array}{cc} \tilde{\Psi}_{ijsv}^{(\ell)} & * \\ \bar{F} & -Z_2 \end{array} \right] < 0, \quad i \in \mathcal{I}_1, \quad (73)$$

where

$$\left\{ \begin{array}{l} \tilde{\Psi}_{ijsv}^{(\ell)} := \text{Sym}\{\bar{\Lambda}_6^{(\ell)} \bar{F}\} + \bar{\Lambda}_{1is} P_{jv} \bar{\Lambda}_{1is}^T - \bar{\Lambda}_2 P_{is} \bar{\Lambda}_2^T \\ \quad + \bar{\Lambda}_3 \mathcal{R} \bar{\Lambda}_3^T + \bar{\Lambda}_{4s}^T \mathcal{Z}_0 \bar{\Lambda}_{4s} - \bar{\Lambda}_5 \mathcal{Z}_1 \bar{\Lambda}_5^T \\ \quad + \bar{\lambda}_i \bar{\Lambda}_7 \mathcal{Q}_i \bar{\Lambda}_7^T + \bar{\mathcal{L}}_{is}^T \bar{\mathcal{L}}_{is} - \gamma^2 \bar{E}_8 \bar{E}_8^T, \\ \tilde{\Lambda}_{1is} := \begin{bmatrix} \tilde{\mathbf{A}}_{is}^T & \bar{E}_5 - \bar{E}_3 & \bar{E}_6 + \bar{E}_7 - \bar{E}_2 - \bar{E}_4 \end{bmatrix}, \\ \tilde{\mathbf{A}}_{is} := \begin{bmatrix} A_s & \mathbf{0} \\ B_{fi} C_s & A_{fi} \end{bmatrix} \begin{bmatrix} A_{ds} \\ B_{fi} C_{ds} \end{bmatrix} \\ \quad \mathbf{0}_{2n_x \times 5n_x} \begin{bmatrix} B_s \\ B_{fi} D_s \end{bmatrix} \begin{bmatrix} a_s \\ a_{fi} \end{bmatrix} \\ \tilde{\mathcal{L}}_{is} := \begin{bmatrix} L_s - D_{fi} C_s & -C_{fi} & L_{ds} - D_{fi} C_{ds} & \mathbf{0}_{n_z \times 5n_x} & -D_{fi} D_s & \mathbf{0}_{n_z \times 1} \end{bmatrix}, \end{array} \right. \quad (74)$$

and the other notations are defined in (71).

Then, let Lyapunov matrices P_{is} in (73) be partitioned as

$$P_{is} := \begin{bmatrix} P_{is(1)} & * \\ P_{is(2)}^T & P_{is(3)} \end{bmatrix}, \quad (75)$$

where $\{P_{is(1)}, P_{is(3)}\} \in \mathbb{S}^{2n_x}$ and $P_{is(2)} \in \mathbb{R}^{2n_x \times 2n_x}$. Considering the similar relaxation inequality given in (53), and by the Schur complement, the subsequent inequality implies

(73):

$$\begin{bmatrix} -P_{jv(1)}^{-1} & * & * & * \\ \mathcal{A}_{is}^T & \bar{\Psi}_{ijsv}^{(\ell)} & * & * \\ \mathbf{0} & \bar{F} & -Z_2 & * \\ \mathbf{0} & \bar{\mathcal{L}}_{is} & \mathbf{0} & -\mathbf{I} \end{bmatrix} < 0, \quad (76)$$

$i \in \mathcal{I}_1, (i, j) \in \Omega, s, v \in \mathcal{S}(i), \ell = 1, 2,$

where

$$\begin{cases} \bar{\Psi}_{ijsv}^{(\ell)} := \text{Sym}\{\bar{\Lambda}_6^{(\ell)} \bar{F}\} + \bar{\Lambda}_{1is} \bar{P}_{jv} \bar{\Lambda}_{1is}^T - \bar{\Lambda}_2 P_{is} \bar{\Lambda}_2^T \\ \quad + \bar{\Lambda}_3 \mathcal{R} \bar{\Lambda}_3^T + \bar{\Lambda}_{4is}^T Z_0 \bar{\Lambda}_{4is} - \bar{\Lambda}_5 Z_1 \bar{\Lambda}_5^T \\ \quad + \bar{\lambda}_i \bar{\Lambda}_7 Q_i \bar{\Lambda}_7^T + \bar{\mathcal{L}}_{is}^T \bar{\mathcal{L}}_{is} - \gamma^2 \bar{E}_8 \bar{E}_8^T, \\ \bar{P}_{jv} := \begin{bmatrix} \mathbf{0} & * \\ P_{jv(2)}^T & P_{jv(3)} \end{bmatrix}. \end{cases} \quad (77)$$

To linearize the matrix inequality, carrying out a congruent transformation to (76) by $\text{diag}\{\tilde{G}_{jv}, \mathbf{I}_{(8n_x+n_w+1)}\}$, it follows from

$$(P_{jv(1)} - \tilde{G}_{jv})^T P_{jv(1)}^{-1} (P_{jv(1)} - \tilde{G}_{jv}) \geq 0 \quad (78)$$

that

$$\tilde{G}_{jv}^T P_{jv(1)}^{-1} \tilde{G}_{jv} \leq P_{jv(1)} - \text{Sym}\{\tilde{G}_{jv}\}. \quad (79)$$

Obviously, based on (79), we arrive at the following inequality implying (76),

$$\begin{bmatrix} P_{jv(1)} - \text{Sym}\{\tilde{G}_{jv}\} & * & * & * \\ \mathcal{A}_{is}^T \tilde{G}_{jv}^T & \bar{\Psi}_{ijsv}^{(\ell)} & * & * \\ \mathbf{0} & \bar{F} & -Z_2 & * \\ \mathbf{0} & \bar{\mathcal{L}}_{is} & \mathbf{0} & -\mathbf{I} \end{bmatrix} < 0, \quad (80)$$

$i \in \mathcal{I}_1, (i, j) \in \Omega, s, v \in \mathcal{S}(i), \ell = 1, 2.$

Then, taking the similar manipulations provided in the proof of Theorem 3.3, one can prescribe the slack variable \tilde{G}_{jv} and the Lyapunov function $P_{is(2)}$ of the following forms, respectively,

$$\begin{cases} \tilde{G}_{jv} := \begin{bmatrix} \tilde{G}_{jv(1)} & \tilde{G}_3 \\ \tilde{G}_{jv(2)} & \tilde{G}_3 \end{bmatrix}, \\ P_{is(2)} := \begin{bmatrix} P_{is(21)} & P_{is(22)} \\ \rho_1 \tilde{G}_3^T & \rho_2 \tilde{G}_3^T \end{bmatrix}. \end{cases} \quad (81)$$

Now, substituting the slack variable \tilde{G}_{jv} and the Lyapunov function $P_{is(2)}$ into (80) and defining the following matrices

$$\tilde{A}_{fi} := G_{1i(3)} A_{fi}, \quad \tilde{B}_{fi} := G_{1i(3)} B_{fi}, \quad \tilde{a}_{fi} := G_{1i(3)} a_{fi}, \quad (82)$$

one can readily obtain (69).

In addition, since $G_{1i(3)}$ is nonsingular as implied by (69), the filter gains can be easily solved by (72). The proof is thus completed. ■

IV. SIMULATION STUDIES

Example 4.1 Consider a T-S fuzzy-affine time-delay system with the form (1), whose system parameters are characterized in (83).

The membership functions are shown in Fig. 1 with $\xi(k) = x_1(k)$. By the decomposition method employed in Section II, we divide the state space into three subspaces: $\mathcal{S}_1 := \{x \in \mathbb{R}^2 | -\varphi_2 \leq x_1 \leq -\varphi_1\}$, $\mathcal{S}_2 := \{x \in \mathbb{R}^2 | -\varphi_1 \leq x_1 \leq \varphi_1\}$, $\mathcal{S}_3 := \{x \in \mathbb{R}^2 | \varphi_1 \leq x_1 \leq \varphi_2\}$, where φ_1 and φ_2 are selected

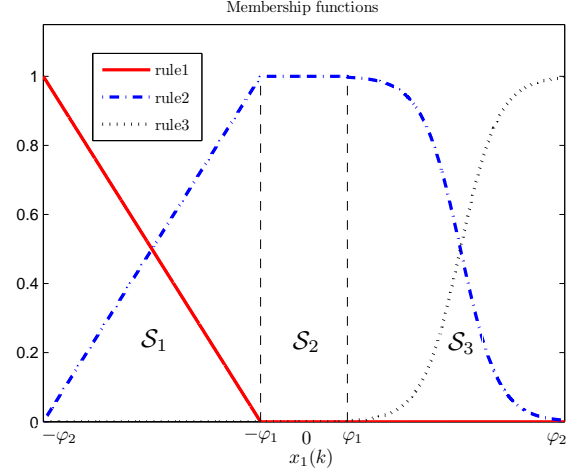


Fig. 1. Membership functions in Examples 4.1 and 4.2

to be $\varphi_1 = 5$ and $\varphi_2 = 30$, respectively. Then, the ellipsoid subspaces can be precisely described by

$$\mathcal{E}_i := \{x \in \mathbb{R}^2 | \|Q_i x + q_i\| \leq 1\}, \quad i = 1, 2, 3 \quad (84)$$

with

$$\begin{aligned} Q_1 &= \begin{bmatrix} \frac{2}{\varphi_2 - \varphi_1} & 0 \end{bmatrix}, \quad q_1 = \frac{\varphi_2 + \varphi_1}{\varphi_2 - \varphi_1} \\ Q_2 &= \begin{bmatrix} \frac{1}{\varphi_1} & 0 \end{bmatrix}, \quad q_2 = 0 \\ Q_3 &= \begin{bmatrix} \frac{2}{\varphi_2 - \varphi_1} & 0 \end{bmatrix}, \quad q_3 = \frac{\varphi_2 + \varphi_1}{\varphi_1 - \varphi_2}. \end{aligned} \quad (85)$$

With the subspaces described in Fig. 1, we can conclude $\mathcal{S} = \{1, 2, 3\}$, $\mathcal{S}(1) = \{1, 2\}$, $\mathcal{S}(2) = \{2\}$, $\mathcal{S}(3) = \{2, 3\}$. Obviously, \mathcal{S}_2 is a crisp subspace, and \mathcal{S}_1 and \mathcal{S}_3 are fuzzy subspaces.

The purpose hereof is to design a piecewise-affine filter (13) for the above system with an \mathcal{H}_∞ performance γ . Choosing $d_1 = 3$ and $d_2 = 11$ of time-delay, it is found that there is no feasible solution by the methods proposed in [40]–[42]. While applying Theorem 3.3 with $\delta_1 = \delta_2 = 0$, one indeed attains the \mathcal{H}_∞ performance $\gamma_{\min} = 7.4152$.

Furthermore, by employing Theorem 3.3 with $\delta_1 = \delta_2 = 0$ and Corollary 3.3 with $\rho_1 = \rho_2 = 0$, a detailed comparison of the attained optimal \mathcal{H}_∞ performance indexes γ_{\min} by the piecewise-affine/piecewise-linear ($a_{fi} \equiv 0, i \in \mathcal{S}$) filters with different synthesis approaches is listed in Table I.

By inspection of Table I, it can be easily observed that the filter design conditions presented in Theorem 3.3 are generally less conservative than those presented in Corollary 3.3. It can also be inspected from Table I that the piecewise-affine filter can achieve better performance than the piecewise-linear one.

To further confirm the effectiveness of filter synthesis conditions in Theorem 3.3, in the sequel we provide another simulation example.

Example 4.2 Consider an inverted pendulum system borrowed from [29], where one damping term is introduced to guarantee the system stability and the angular velocity is subject to delay. The dynamics of the pendulum can be

$$\begin{aligned}
\begin{bmatrix} A_1 & A_{d1} & B_1 & a_1 \\ C_1 & C_{d1} & D_1 & \\ L_1 & L_{d1} & & \end{bmatrix} &= \begin{bmatrix} 0.675 & 0.225 & -0.1 & 0.07 & 0 & 0 \\ 0.075 & 0.675 & -0.07 & -0.07 & 0.5 & 0.6 \\ 0.8 & 0 & 0.2 & 0 & 1 & \\ 1 & 0 & 0 & 0.2 & & \end{bmatrix}, \\
\begin{bmatrix} A_2 & A_{d2} & B_2 & a_2 \\ C_2 & C_{d2} & D_2 & \\ L_2 & L_{d2} & & \end{bmatrix} &= \begin{bmatrix} 0.45 & 0.1 & 0.09 & 0 & 0.4 & 0 \\ 0.1 & 0.2 & -0.07 & 0.05 & 0.5 & 0 \\ 0.8 & 0 & 0.2 & 0 & 0.5 & \\ 1 & 0 & 0 & 0.2 & & \end{bmatrix}, \\
\begin{bmatrix} A_3 & A_{d3} & B_3 & a_3 \\ C_3 & C_{d3} & D_3 & \\ L_3 & L_{d3} & & \end{bmatrix} &= \begin{bmatrix} 0.675 & 0.3 & -0.1 & 0.07 & -0.5 & 0 \\ -0.075 & 0.675 & -0.07 & -0.07 & 0 & -0.6 \\ 0.8 & 0 & 0.2 & 0 & 0.5 & \\ 1 & 0 & 0 & 0.2 & & \end{bmatrix}.
\end{aligned} \tag{83}$$

TABLE I
COMPARISON OF MINIMUM \mathcal{H}_∞ PERFORMANCE FOR DIFFERENT CASES

Methods	Filter Model	$d_1 = 2$		$d_1 = 3$		$d_1 = 5$		$d_1 = 8$		$d_1 = 10$	
		$d_2 = 7$	$d_2 = 10$	$d_2 = 7$	$d_2 = 11$	$d_2 = 9$	$d_2 = 12$	$d_2 = 12$	$d_2 = 14$	$d_2 = 14$	$d_2 = 16$
Theorem 3.3	Affine model	1.8868	4.2163	1.7664	7.4152	1.8743	4.3366	1.9958	4.1008	2.0340	6.6900
	Linear model	1.9116	4.5333	1.7844	7.7072	1.9007	4.8697	2.0146	7.0017	2.0865	7.9496
Corollary 3.3	Affine model	3.7206	∞	2.4579	∞	2.7844	∞	2.8869	∞	2.9196	∞
	Linear model	4.7978	∞	2.8007	∞	2.9967	∞	3.0020	∞	3.0987	∞

described by

$$\begin{cases} \dot{x}_1(t) = \lambda x_2(t) + (1 - \lambda)x_2(t - d(t)) \\ \dot{x}_2(t) = -\frac{g \sin(x_1(t)) + \frac{2b}{m}[\lambda x_2(t) + (1 - \lambda)x_2(t - d(t))]}{\frac{4}{3}l - aml \cos^2(x_1(t))} \\ \quad - \frac{\frac{aml}{2}[\lambda x_2(t) + (1 - \lambda)x_2(t - d(t))]^2 \sin(2x_1(t))}{\frac{4}{3}l - aml \cos^2(x_1(t))} \\ \quad - \frac{a \cos(x_1(t))w(t)/2}{\frac{4}{3}l - aml \cos^2(x_1(t))} \\ y(t) = \sin(x_1(t)) + \lambda x_2(t) + (1 - \lambda)x_2(t - d(t)) + w(t) \\ z(t) = x_1(t) \end{cases} \tag{86}$$

where x_1 represents the angle of the pendulum along the vertical; x_2 denotes the angular velocity; $g = 9.8\text{m/s}^2$ stands for the gravity constant; $m = 2\text{kg}$ denotes the mass of the pendulum, $a = 1/(m + M)$; $M = 8\text{kg}$ denotes the mass of the cart; $2l = 1\text{m}$ denotes the length of the pendulum; $b = 0.7\text{Nm/s}$ denotes the damping coefficient; and $w(t)$ represents the exogenous disturbance.

By linearizing and discretizing procedure for the nonlinear plant at the origin and $x = (\pm\pi/3, 0)$ with the sampling period $T = 0.05\text{s}$, respectively, one attains a discrete-time T-S fuzzy-affine model with state-delay:

Plant Rule \mathcal{F}^l : IF $x_1(k)$ is F^l , THEN

$$\begin{cases} x(k+1) = A_l x(k) + A_{dl} x(k - d(k)) + a_l + B_l w(k) \\ y(k) = C_l x(k) + C_{dl} x(k - d(k)) + D_l w(k) \\ z(k) = L_l x(k), \quad l \in \{1, 2, 3\} \end{cases} \tag{87}$$

where the system matrices are defined in (88).

The membership functions are shown in Fig. 1. By space division presented in the previous section, we attain the following three subspaces $\mathcal{S}_1 := \{x \in \mathbb{R}^2 | -\varphi_2 \leq x_1 \leq -\varphi_1\}$, $\mathcal{S}_2 := \{x \in \mathbb{R}^2 | -\varphi_1 \leq x_1 \leq \varphi_1\}$, $\mathcal{S}_3 := \{x \in \mathbb{R}^2 | \varphi_1 \leq x_1 \leq \varphi_2\}$, where φ_1 and φ_2 are chosen to be $\varphi_1 = \frac{\pi}{3}$ and $\varphi_2 = \frac{\pi}{2}$, respectively. The three ellipsoid subspaces can be

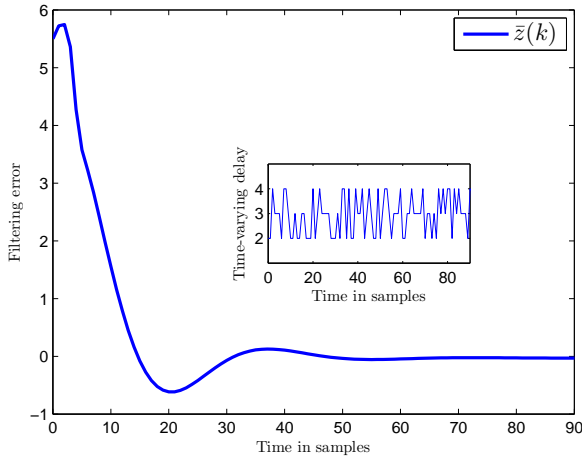
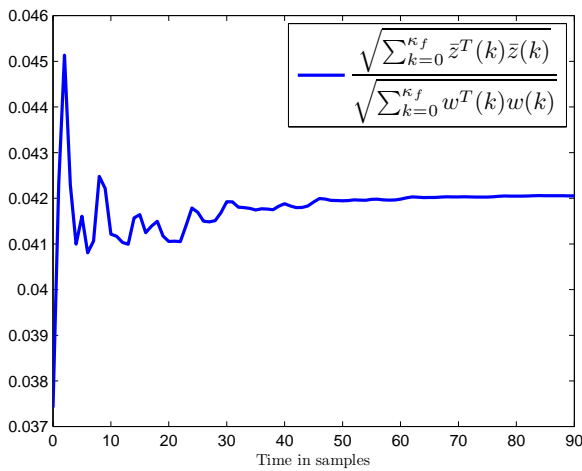
accurately described by (84) and (85). In this example, the time-delay $d(k)$ satisfies (2) with $d_1 = 2$ and $d_2 = 4$, and $w(k) = 0.1e^{-0.04k} \sin(k)$.

The purpose hereof is to synthesize a piecewise-affine filter (12) that guarantees the asymptotic stability of the resulting FES with an \mathcal{H}_∞ performance γ_{\min} . By virtue of Theorem 3.3 with $\delta_1 = \delta_2 = 0$, one attains the feasible solution of $\gamma_{\min} = 0.0557$ with the filter parameters,

$$\begin{aligned}
&\begin{bmatrix} A_{f1} & B_{f1} & a_{f1} \\ C_{f1} & D_{f1} & \end{bmatrix} \\ &= \begin{bmatrix} 0.9082 & 0.0273 & -0.0005 & -0.0483 \\ -0.9887 & 0.8870 & 0.0390 & -2.0307 \\ -1.9943 & 0 & 3.4381 \times 10^{-5} & \end{bmatrix}, \\ &\begin{bmatrix} A_{f2} & B_{f2} \\ C_{f2} & D_{f2} \end{bmatrix} \\ &= \begin{bmatrix} 0.9026 & 0.0191 & -0.1926 \\ -0.9748 & 0.7046 & 0.0100 \\ -2.0501 & -0.0304 & 0.0014 \end{bmatrix}, \\ &\begin{bmatrix} A_{f3} & B_{f3} & a_{f3} \\ C_{f3} & D_{f3} & \end{bmatrix} \\ &= \begin{bmatrix} 0.9082 & 0.0273 & -0.0005 & 0.0482 \\ -0.9892 & 0.8870 & 0.0090 & 2.0322 \\ -1.9944 & 0.0001 & 2.5591 \times 10^{-5} & \end{bmatrix}.
\end{aligned}$$

To further verify the effectiveness of the developed results, simulations are performed with the initial condition $\bar{x}(0) = [1.5 \ 1.5 \ 0 \ 0]^T$. With a stochastically generated time-varying delay between the lower bound $d_1 = 2$ and upper bound $d_2 = 4$, Fig. 2 shows the time response of the estimation error $\bar{z}(k)$, and Fig. 3 depicts the time response of the ratio $\sqrt{\sum_{k=0}^{\kappa_f} \bar{z}^T(k)\bar{z}(k)} / \sqrt{\sum_{k=0}^{\kappa_f} w^T(k)w(k)}$ under zero initial conditions. It can be inspected from Fig. 3 that the time response of ratio $\sqrt{\sum_{k=0}^{\kappa_f} \bar{z}^T(k)\bar{z}(k)} / \sqrt{\sum_{k=0}^{\kappa_f} w^T(k)w(k)}$ is

$$\begin{aligned}
\begin{bmatrix} A_1 & A_{d1} & B_1 & a_1 \\ C_1 & C_{d1} & D_1 & \\ L_1 & & & \end{bmatrix} &= \begin{bmatrix} 1 & 0.045 & 0 & 0.005 & 0 & 0 \\ -0.6315 & 0.8108 & 0 & 0.0891 & -0.0019 & 0.9675 \\ 0.8270 & 0.9 & 0 & 0.1 & 1 & \end{bmatrix}, \\
\begin{bmatrix} A_2 & A_{d2} & B_2 & a_2 \\ C_2 & C_{d2} & D_2 & \\ L_2 & & & \end{bmatrix} &= \begin{bmatrix} 1 & 0 & 0 & 0.005 & 0 & 0 \\ -0.8558 & 0.7894 & 0 & 0.0877 & -0.0039 & 0 \\ 0.8270 & 0.9 & 0 & 0.1 & 1 & \end{bmatrix}, \\
\begin{bmatrix} A_3 & A_{d3} & B_3 & a_3 \\ C_3 & C_{d3} & D_3 & \\ L_3 & & & \end{bmatrix} &= \begin{bmatrix} 1 & 0 & 0 & 0.005 & 0 & 0 \\ -0.6315 & 0.8108 & 0 & 0.0891 & -0.0019 & -0.9675 \\ 0.8270 & 0.9 & 0 & 0.1 & 1 & \end{bmatrix}.
\end{aligned} \tag{88}$$

Fig. 2. Response of the estimation error $\bar{z}(k)$ in Example 4.2Fig. 3. Response of the ratio $\sqrt{\sum_{k=0}^{\kappa_f} \bar{z}^T(k)\bar{z}(k)} / \sqrt{\sum_{k=0}^{\kappa_f} w^T(k)w(k)}$ in Example 4.2

less than 0.05, which is obviously less than the calculated disturbance attenuation level 0.0557, and thus the designed filter achieves satisfactory performance.

V. CONCLUSIONS

This paper has proposed a novel approach to addressing the delay-dependent piecewise-affine \mathcal{H}_∞ filtering problem for discrete-time T-S fuzzy-affine systems with time-varying delay. Based on an appropriate piecewise-fuzzy LKF, combined with a new summation inequality, improved reciprocally convex inequality and S-procedure, the bounded real lemma has been firstly derived for the underlying fuzzy-affine system. Furthermore, the piecewise-affine filter synthesis has been carried out. It has been shown that the existence of desired filter parameters can be checked by the feasibility of a set of LMIs. Finally, simulation examples have been presented to validate the effectiveness and less conservatism of the developed approaches. Notice that the generalizations of the developed approaches to the state estimation of continuous-time case deserve further study. Another interesting future research topic is the investigation on the non-synchronous filter design for the networked T-S fuzzy-affine dynamic systems in the piecewise homogenous polynomial-Lyapunov-functions framework.

ACKNOWLEDGMENTS

The authors are grateful to the associate editor and anonymous reviewers for their constructive comments based on which the presentation of this paper has been greatly improved.

APPENDIX

Lemma A1 (Reciprocally convex inequality). [47] Given two matrices $Z_1 \in \mathbb{S}^{n_1}$ and $Z_2 \in \mathbb{S}^{n_2}$, if there exists a matrix $X \in \mathbb{R}^{n_1 \times n_2}$ such that $\begin{bmatrix} Z_1 & X \\ X^T & Z_2 \end{bmatrix} \geq 0$, then the inequality

$$\begin{bmatrix} \frac{1}{\alpha} Z_1 & \mathbf{0} \\ \mathbf{0} & \frac{1}{1-\alpha} Z_2 \end{bmatrix} \geq \begin{bmatrix} Z_1 & X \\ X^T & Z_2 \end{bmatrix}$$

holds with any scalar $\alpha \in (0, 1)$.

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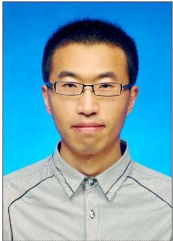
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